

8 Gauss' Law

$$\Phi \equiv \oint_S \vec{E} \cdot \hat{n} \, dA$$

$$\Phi = \frac{Q_{enc}}{\epsilon_0}$$

Cylinder:

$$dV = r \, dr \, d\phi \, dz$$

$$dV_{sym} = 2\pi r \, dr \, dz$$

Sphere:

$$dV = \rho^2 \sin \theta \, d\theta \, d\phi \, d\rho$$

$$dV_{sum} = 4\pi \rho^2 \, d\rho$$

29-1 A spherical rubber balloon carries a charge that is uniformly distributed over its surface. As the balloon is blown up, how does \vec{E} vary for points (a) inside the balloon, (b) at the surface of the balloon, and (c) outside the balloon?

29-2 A cube with side length L is oriented with one corner at the origin, and its edges along the positive x, y, z axes. Find the electric flux through the xy (bottom) face of the cube, in the presence of a uniform electric field \vec{E} , in N/C, (a) $6\hat{k}$ (b) $-2\hat{j}$ (c) $-3\hat{i} + 4\hat{k}$. (d) Calculate the total flux through the cube for each case.

29-3 An electric field passes through a hemisphere with a flat base of radius R . The electric field is uniform, $\vec{E} = (E_x, E_y, E_z)$. The axis of the hemisphere is in the $+z$ direction. Calculate the electric flux through the curved (top) part of the hemisphere.

29-6 The net electric flux through each face of a 6-sided dice has magnitude in units of $10^3 \text{ N m}^2/\text{C}$ equal to the number N of spots on the face (1 through 6). The flux is inward for N odd and outward for N even. What is the net charge inside the dice?

29-12 A point charge q is placed at one corner of a cube of edge a . What is the flux through each of the cube faces? (*Hint*: Use Gauss law and symmetry arguments.)

29-32 A large flat nonconducting surface carries a uniform charge density σ . A small circular hole of radius R has been cut in the middle of the sheet. Calculate the electric field at point P , a distance z from the center of the hole along its axis.

29-41 Positive charge is distributed uniformly throughout a long, nonconducting cylindrical shell of inner radius R and outer radius $2R$. At what radial depth beneath the outer surface of the charge distribution is the electric field strength equal to one-half the surface value?

29-42 The spherical region $a < r < b$ carries a charge per unit volume of $\rho = A/r$, where A is a constant. At the center ($r = 0$) of the enclosed cavity is a point charge q . What should be the value of q so that the electric field in the region $a < r < b$ has constant magnitude?

29-44 A spherical region carries a uniform charge per unit volume ρ . Let \vec{r} be the vector from the center of the sphere to a general point P within the sphere. (a) Show that the electric field at P is given by $\vec{E} = \rho\vec{r}/3\epsilon_0$. (b) A spherical cavity is created in the above sphere, not necessarily in the center. Using superposition concepts, show that the electric field at all points within the cavity is $\vec{E} = \rho\vec{a}/3\epsilon_0$ (uniform field), where \vec{a} is the vector connecting the center of the sphere with the center of the cavity.

29-46 A plane slab of thickness d has a uniform volume charge density ρ . Find the magnitude of the electric field at all points in space both (a) inside and (b) outside the slab, in terms of x , the distance measured from the median plane of the slab.