7 Continuous charge distributions

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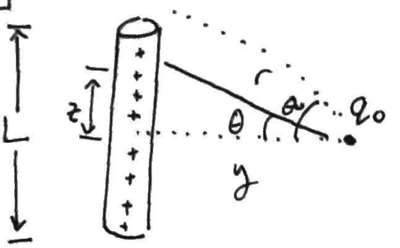


Split rod into $dq = \lambda\, dl = \lambda\, dz = \dfrac{\lambda y\, d\theta}{\cos^2\theta}$

$$\frac{z}{y} = \tan\theta \rightarrow dz = \frac{y\, d\theta}{\cos^2\theta}$$

$$r = \sqrt{y^2 + z^2} = y\sqrt{1+\tan^2\theta} = \sout{y\cos\theta}$$

$$= y\sqrt{\sec^2\theta} = \frac{y}{\cos\theta}$$

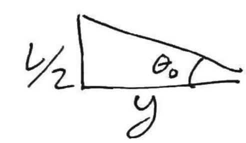
Magnitude of $d\vec{E}$ from each dq $\Rightarrow$ $|d\vec{E}| = \dfrac{1}{4\pi\varepsilon_0}\dfrac{dq}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\lambda y\, d\theta}{\cos^2\theta}\dfrac{\cos^2\theta}{y^2}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{y}\, d\theta$$

$z \rightarrow -z$ reflection symmetry $\Rightarrow$ only $\hat{y}$ component survives.

$$dE_y = |d\vec{E}|(\hat{r}\cdot\hat{y}) = |d\vec{E}|\cos\theta$$

Superposition $\Rightarrow$ $E_y = \displaystyle\int dE_y = \int_{-\theta_0}^{+\theta_0} \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{y}\cos\theta\, d\theta$



$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{y}\left[\sin\theta\right]_{-\theta_0}^{\theta_0}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{y}\left(\sin\theta_0 - \sin(-\theta_0)\right) = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{y}\left(2\sin\theta_0\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{y}\, 2\, \frac{L/2}{\sqrt{y^2 + L^2/4}}$$

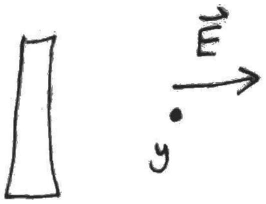
$$\boxed{E_y = \frac{1}{4\pi\varepsilon_0}\frac{\lambda L}{y\sqrt{y^2 + L^2/4}}}$$

(Check: $y \gg L \Rightarrow \dfrac{\lambda L}{y^2} \sim \dfrac{Q}{y^2}$

$y \ll L \Rightarrow \dfrac{\lambda L}{y(L/2)} = \dfrac{2\lambda}{y}$)

These agree w/ point charge & infinite line, respectively

(a) So at point y, $\vec{E} = E_y \hat{y} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda L}{y\sqrt{y^2 + L^2/4}} \hat{y}$



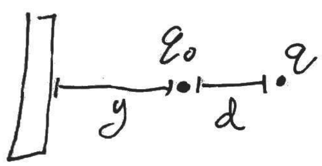
To cancel this off, need

$$\vec{E}_q = -\frac{1}{4\pi\varepsilon_0} \frac{\lambda L}{y\sqrt{y^2+L^2/4}} \hat{y}$$

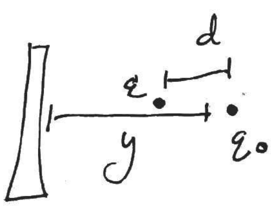
If $q > 0$, then , we want q to right of q_0.

Distance d s.t. $\frac{1}{4\pi\varepsilon_0} \frac{\lambda L}{y\sqrt{y^2+L^2/4}} = \frac{1}{4\pi\varepsilon_0} \frac{q}{d^2}$

$$\Rightarrow \boxed{d = \sqrt{q/\lambda L} \cdot \left(y^4 + L^2 y^2/4\right)^{1/4}}$$



(b) If $q < 0$, then same but to the left:



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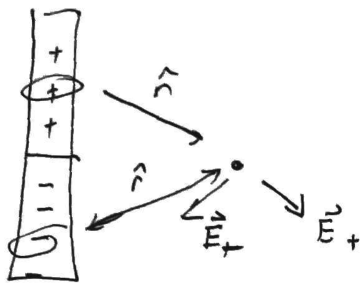
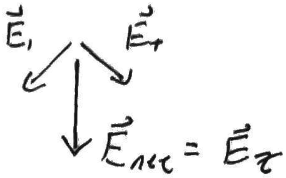
Setup is the same as the previous problem, but now at this point the E_y from top/bottom half of rod will cancel, and E_z remains:





$$\Rightarrow \quad |d\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{y}\,d\theta$$

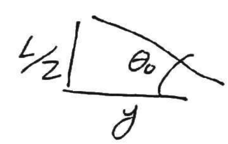
$$dE_z = \pm|d\vec{E}|\,(\hat{r}\cdot\hat{z}) = \mp|d\vec{E}|\sin\theta$$

$+$ if $\theta > 0$ $(\lambda > 0)$ $\quad$ $+$ if $\theta < 0$

$-$ if $\theta < 0$ $(\lambda < 0)$ $\quad$ $-$ if $\theta > 0$

Superposition

$$\Rightarrow \quad E_z = \int dE_z = -2\int_0^{+\theta_0} \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{y}\sin\theta\,d\theta$$



$$= -\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}\left[-\cos\theta\right]_0^{\theta_0}$$

$$= -\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}\left[1-\cos\theta_0\right] = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}\left[1-\frac{y}{\sqrt{y^2+L^2/4}}\right]$$

$$= -\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y} + \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{\sqrt{y^2+L^2/4}}$$

$$\Rightarrow \quad \boxed{\vec{F} = \vec{E}q_0 = \left(-\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y} + \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{\sqrt{y^2+L^2/4}}\right)q_0\,\hat{z}}$$

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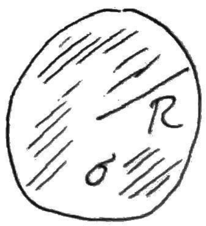
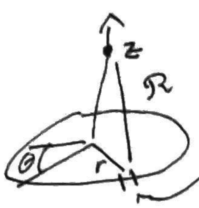


Superposition:  $= \int_0^R dr$ 

$$\lambda = \sigma\, dr \qquad \left(dq = \sigma\, dA = \sigma\, dr\, dl = \lambda\, dl\right)$$

For each ring:



$$dq = \lambda\, dl = \lambda\, r\, d\theta$$

$$\mathfrak{r}^2 = z^2 + r^2 \Rightarrow |dE| = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\mathfrak{r}^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda r\, d\theta}{z^2+r^2}$$

Rotation / reflection symmetry
$\Rightarrow$ Only dE_z remains.

$$dE_z = |dE|\, \hat{z}\cdot\hat{\mathfrak{r}} = |dE|\,\frac{z}{\mathfrak{r}} = \frac{1}{4\pi\varepsilon_0}\frac{z\cdot \lambda r\, d\theta}{(z^2+r^2)^{3/2}}$$

$$\Rightarrow E_{z,\text{ring}} = \int dE_z = \int_0^{2\pi}\frac{1}{4\pi\varepsilon_0}\frac{z\,\lambda r}{(z^2+r^2)^{3/2}}\,d\theta$$

$$= \frac{1}{2\varepsilon_0}\frac{z\,\lambda r}{(z^2+r^2)^{3/2}}$$

$$= \frac{1}{2\varepsilon_0}\frac{z}{(z^2+r^2)^{3/2}}(\sigma\, r\, dr) \qquad \lambda = \sigma\, dr$$

$$\Rightarrow E_{\text{disk}} = \int_0^R \frac{1}{2\varepsilon_0}\frac{z}{(z^2+r^2)^{3/2}}(\sigma\, r\, dr)$$

Substitute: $u = z^2 + r^2 \Rightarrow du = 2\, r\, dr \quad \left(\frac{du}{2} = r\, dr\right)$

$r = 0 \Rightarrow u = z^2$

$r = R \Rightarrow u = z^2 + R^2$

$$\Rightarrow E_{\text{disk}} = \frac{\sigma z}{4\varepsilon_0}\int \frac{du}{u^{3/2}}$$

$$= \frac{\sigma z}{4\varepsilon_0}\int_{z^2}^{z^2+R^2} u^{-3/2}\, du$$

$$\frac{d}{du}u^{-1/2} = -\frac{1}{2}u^{-3/2}$$

$$= -\frac{\sigma z}{2\varepsilon_0}\left[u^{-1/2}\right]_{z^2}^{z^2+R^2}$$

$$= \frac{\sigma z}{2\varepsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right] = \boxed{\frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]\hat{z}}$$

(Check $z \ll R \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$)