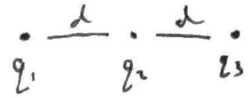
# Electric Charge

**27-7**



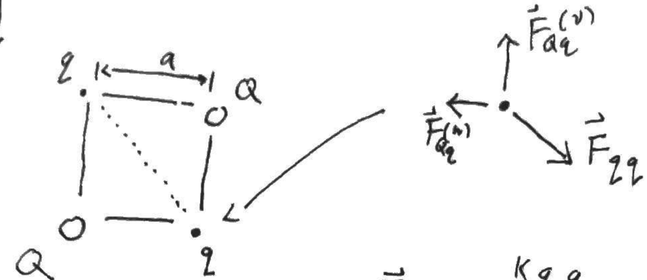

$$0 = F_{13} + F_{23} = \frac{K q_1 q_3}{(2d)^2} + \frac{K q_2 q_3}{d^2} = \frac{K q_3}{d^2}\left(q_1/4 + q_2\right)$$

$$\Rightarrow \boxed{q_1 = -4q_2}$$

(Intuitively, $q_1$ is twice as far, so needs $(2)^2 = 4\times$ the charge to "beat" the $1/r^2$)

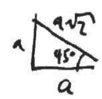
**27-14**

(a)




$F_{Qq}$ should be attractive, to hold $q$ in place.
$\Uparrow$
$\vec{F}_{qq}$ - repulsive (like charges) $\hookrightarrow Q < 0,\ q > 0$

$$\vec{F}_{qq} = \frac{K q q}{\underbrace{(a\sqrt{2})^2}} \cdot \underbrace{\frac{\hat{x} - \hat{y}}{\sqrt{2}}}_{\text{normalized unit vector}}$$




$$\vec{F}^{(h)}_{Qq} = K\frac{Qq}{a^2}\,\hat{x}$$

$$\vec{F}^{(v)}_{Qq} = K\frac{Qq}{a^2}(-\hat{y})$$

$$\Rightarrow \vec{F}_{net} = \vec{F}_{qq} + \vec{F}^{(h)}_{Qq} + \vec{F}^{(v)}_{Qq} = \frac{Kq}{a^2}\left(\frac{q}{2\sqrt{2}}(\hat{x} - \hat{y}) + Q\hat{x} - Q\hat{y}\right)$$

$$= \frac{Kq}{a^2}\underbrace{\left(\frac{q}{2\sqrt{2}} + Q\right)}_{=0}(\hat{x} - \hat{y}) = 0$$

$$=0 \iff \boxed{Q = -\frac{q}{2\sqrt{2}}}$$

(b) No.

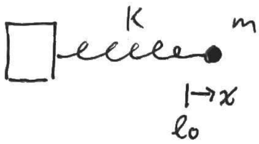
- "By symmetry" - rotate square 90° and then do same calculation - get $q = -\frac{Q}{2\sqrt{2}}$

$\hookrightarrow q = Q = 0$ only solution to both

~~Can also treat diff. but superposition~~

27-22

First recall spring version:



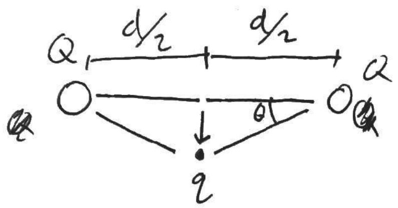

$$F = -Kx$$

$$m\ddot{x} = -Kx \rightarrow \ddot{x} = \frac{-K}{m}x$$

$$x = \sin\omega t \rightarrow \dot{x} = \omega\cos\omega t, \quad \ddot{x} = -\omega^2 \sin\omega t = -\omega^2 x$$

$$\omega = \sqrt{K/m}, \quad \text{period } T = \frac{1}{f} = \frac{2\pi}{\omega}$$

↳ Basically, need to find "spring const." $K_{eff}$




The horizontal components of the $\vec{F}_{Qq}$ will cancel, so just need to figure out vertical:

$$|F_{Qq,y}| = \sin\theta\,|F_{Qq}| \underset{\substack{\uparrow \\ \text{small angle} \sim \text{small oscillation}}}{\cong} \frac{y}{(d/2)}|F_{Qq}|$$

$$|F_{Qq}| = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{\sqrt{d^2/4 + y^2}^{\,2}}$$

$$\underset{\substack{\uparrow \\ y \text{ small}}}{\cong} \frac{1}{4\pi\varepsilon_0}\frac{Qq}{(d/2)^2}$$

$$\Rightarrow F_{tot} = -2\cdot\frac{y}{(d/2)}\cdot\frac{1}{4\pi\varepsilon_0}\frac{Qq}{(d/2)^2}\hat{y} \equiv -K_{eff}\,y\,\hat{y}$$

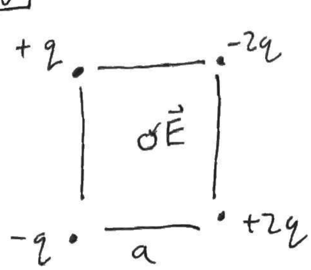
$$\Rightarrow K_{eff} = \left(2\cdot\frac{1}{d/2}\cdot\frac{1}{4\pi\varepsilon_0}\frac{Qq}{(d/2)^2}\right)$$

$$= \frac{16Qq}{d^3\cdot 4\pi\varepsilon_0} = \frac{4Qq}{\pi\varepsilon_0 d^3}$$

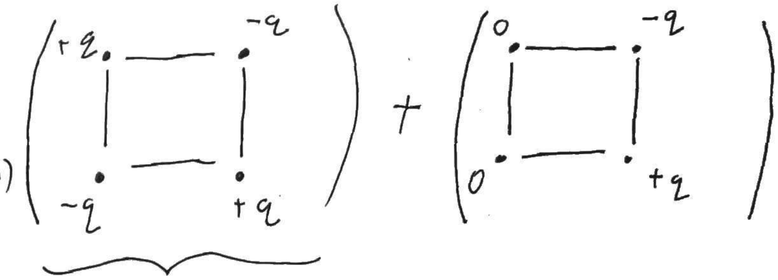
$$\hookrightarrow 1/\omega = \sqrt{m/K_{eff}} = \sqrt{\frac{\pi\varepsilon_0 d^3 m}{4Qq}}$$

$$\boxed{T = 2\pi\cdot\frac{1}{\omega} = \left(\frac{\pi^3\varepsilon_0 d^3 m}{Qq}\right)^{1/2}}$$

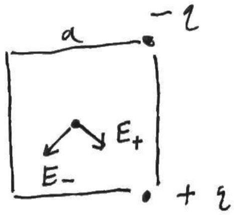
28-8



= (superposition)



$$\vec{E}_{center} = 0$$

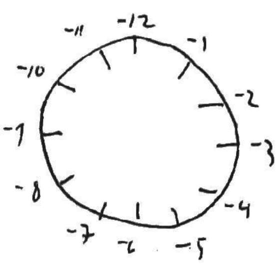


$$\vec{E}_+ = \frac{Kq}{(a/\sqrt{2})^2} \cdot \frac{\hat{x}-\hat{y}}{\sqrt{2}} = \frac{Kq\sqrt{2}}{a}(\hat{x}-\hat{y})$$

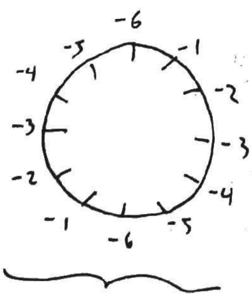
$$\vec{E}_- = \frac{Kq}{(a/\sqrt{2})^2} \cdot \frac{-\hat{x}-\hat{y}}{\sqrt{2}} = \frac{Kq\sqrt{2}}{a}(-\hat{x}-\hat{y})$$

$$\Rightarrow \vec{E}_{tot} = \vec{E}_+ + \vec{E}_- = \boxed{\frac{2\sqrt{2}\,Kq}{a} \cdot (-\hat{y})}$$

28-9 (My favorite!)



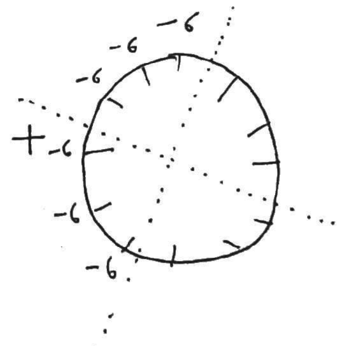
=



$\vec{E}_{center} = 0$

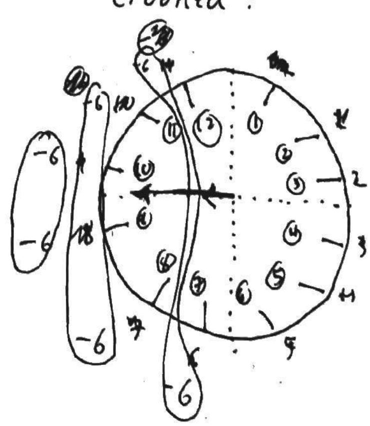
(pair opposite charges)

+



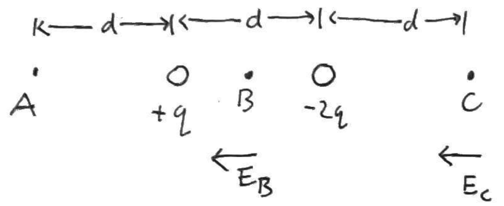
↳ Imagine hanging the clock slightly crooked:

Now pair the charges −6 @ 9+10, 8+11, 7+12



↳ Each pair gives an electric field pointing to $\boxed{9:30}$

28-22

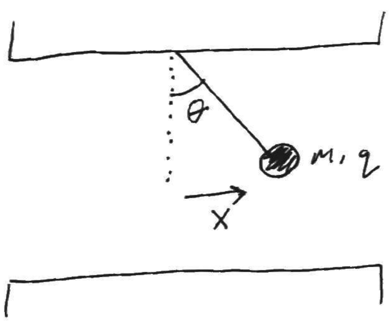



$$E_A = \left(+\frac{kq}{d^2} - \frac{2kq}{(2d)^2}\right)\hat{x} = \frac{kq}{d^2}\left(1-\frac{1}{4}\right)\hat{x} = \frac{3}{4}\frac{kq}{d^2}\hat{x}$$

$$E_B = \left(\frac{kq}{(d/2)^2} + \frac{(-2kq)}{(d/2)^2}\right)\hat{x} = \frac{kq}{d^2}(4-8)\hat{x} = -4\frac{kq}{d^2}\hat{x}$$

$$E_C = \left(\frac{kq}{(2d)^2} + \frac{(-2kq)}{d^2}\right)\hat{x} = \frac{kq}{d^2}\left(\frac{1}{4}-2\right)\hat{x} = -\frac{7}{4}\frac{kq}{d^2}\hat{x}$$

28-44





$\vec{F}_g + \vec{F}_E \equiv \vec{F}$

For the pendulum, the tension will cancel against the radial component of force in-line w/ the string, leaving

$$F_\theta = -F\sin\theta = -(mg \pm qE)\sin\theta \approx -(mg \pm qE)\theta$$

(+ if $qE$ same direction as gravity)

$\cancel{\Rightarrow m\ddot{\theta} = -(mg \pm qE)\theta}$

$\cancel{\hookrightarrow T = 2\pi\sqrt{\phantom{x}}}$

$$F_x = -\frac{mg \pm qE}{L}x$$

$$\Rightarrow ma = m\ddot{x} = -\frac{mg \pm qE}{L}x$$

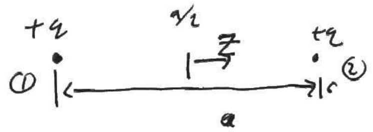
$$\hookrightarrow \boxed{T = 2\pi\sqrt{\frac{mL}{mg \pm qE}}}$$

(a) + + + / $\downarrow E$ / − − − $\rightarrow q\vec{E}$ points same as $\vec{F}_g \rightarrow T = 2\pi\sqrt{\frac{mL}{mg+qE}}$

(b) − − − − / $\uparrow E$ / + + + + $\rightarrow q\vec{E}$ points opposite $\vec{F}_g \rightarrow T = 2\pi\sqrt{\frac{mL}{mg-qE}}$

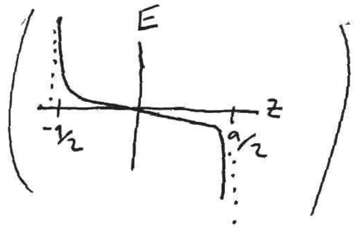
(as long as $mg > qE$)

28-53

(a) 



$$E(z) = \frac{Kq}{d_{①}^2} - \frac{Kq}{d_{②}^2} = \frac{Kq}{(a/2+z)^2} - \frac{Kq}{(a/2-z)^2}$$





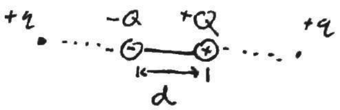
$$\frac{dE}{dz} = -\frac{Kq}{(a/2+z)^3} - (-1)^2\frac{Kq}{(a/2-z)^3} = -Kq\left[\frac{1}{(a/2+z)^3}+\frac{1}{(a/2-z)^3}\right]$$

$$\left.\frac{dE}{dz}\right|_{z=0} = -Kq\left[\frac{1}{(a/2)^3}+\frac{1}{(a/2)^3}\right] = -\frac{16\,Kq}{a}$$

(b) Taylor series around $z=0$: $E(z) = \underbrace{E(0)}_{\frac{Kq}{(a/2)^2}-\frac{Kq}{(a/2)^2}=0} + z\cdot\underbrace{\left.\frac{dE}{dz}\right|_{z=0}}_{\neq 0} + z^2\cdot\underbrace{\left.\frac{dE^2}{dz^2}\right|_{z=0}} + \cdots$

(Also $=0$, since $E(z) = -E(-z)$, but thats just a bonus)

↳ $E(z) = -\frac{16\,Kq}{a}\cdot z + \mathcal{O}(z^3)$





(Intuitively, the ⊕ will be repelled by nearby +q, and ⊖ ~~end~~ attracted by nearby +q.)

$p = Qd.$

$$\vec{F} = +Q\cdot E(d/2) - Q\cdot E(-d/2)$$

$$= Q\left(-\frac{16Kq}{a}\cdot\frac{d}{2}\right) - Q\cdot\left(-\frac{16Kq}{a}\cdot\frac{-d}{2}\right)$$

$$= -Q\left(\frac{16Kq}{a}\frac{d}{2}\cdot 2\right) = -\frac{16\,KqQd}{a} = \boxed{-\left(\frac{16\,Kq}{a}\right)\cdot p} = -\frac{dE}{dz}\cdot p$$

(Pointing ⊖—⊕ ($p>0$) ⇒ Force to left

⊕—⊖ ($p<0$) ⇒ Force to right)