## 5 Entropy and the Second Law of Thermodynamics

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\begin{array}{lll}
\Delta S=\int \frac{\mathrm{d} Q}{T} & \mathrm{~d} Q=T \mathrm{~d} S & \Delta S_{\text {total }} \geq 0 \\
e=\frac{W}{Q_{H}}=1-\frac{Q_{L}}{Q_{H}} & e_{C}=1-\frac{T_{L}}{T_{H}} & e<e_{C}
\end{array}
$$

## 5.a RHK Questions

24-16 You wish to increase the efficiency of a Carnot engine as much as possible. You can do this by increasing $T_{H}$ a certain amount, keeping $T_{L}$ constant, or by decreasing $T_{L}$ the same amount, keeping $T_{H}$ constant. Which would you do?

24-17 We have seen that real engines always discard substantial amounts of heat to their lowtemperature reservoirs. It seems a shame to throw this heat energy away. Why not use this heat to run a second engine, the low-temperature reservoir of the first engine serving as the high-temperature reservoir of the second?

24-21 Give examples in which the entropy of a system decreases and explain why the second law of thermodynamics is not violated.

## 5.b RHK Exercises

24-8 Suppose that the same amount of heat, 200 J , is transferred by conduction from a heat reservoir at a temperature of 400 K to another reservoir, the temperature of which is (a) 100 K , (b) 200 K , (c) 300 K, and (d) 360 K . Calculate the changes in entropy and discuss the trend.

24-9 (Modified) A brass rod is in thermal contact with a heat reservoir at 400 K at one end and a heat reservoir at 300 K at the other end, and has reached a steady state. (a) Sketch the temperature profile of the rod. (b) Compute the total change in the entropy arising from the process of conduction of 1200 J of heat through the rod. (c) Does the entropy of the rod change in the process?

## 5.c RHK Problems

24-1 At very low temperatures, the molar specific heat of many solids is (approximately) proportional to $T^{3}$; that is $C_{V}=A T^{3}$ where $A$ depends on the particular substance. For aluminum, $A=3.15 \times 10^{-5}$ $\mathrm{J} / \mathrm{mol} \mathrm{K}{ }^{4}$. Find the entropy change of 4.8 mol of aluminum when its temperature is raised from 5.0 to 10 K.

24-6 A 12 g ice-cube at $-10^{\circ} \mathrm{C}$ is placed in a lake whose temperature is $+15.0^{\circ} \mathrm{C}$. Calculate the change in entropy of the system as the ice cube comes to thermal equilibrium with the lake. (Hint: Will the ice cube affect the temperature of the lake?)

24-7 A system consists of two objects that are allowed to come into thermal contact. Object 1 has mass $m_{1}$, specific heat capacity $c_{1}$, and is originally at temperature $T_{1, i}$. Object 2 has mass $m_{2}$, specific heat capacity $c_{2}$, and is originally at temperature $T_{2, i}$ As object 1 slowly cools, object 2 slowly warms. (a) Write an expression for the temperature of object $2, T_{2}$, as a function of the temperature of object $1, T_{1}$ (b) Find the change in entropy of the system $S$ as a function of $T_{1}$ (c) Show that $\Delta S$ is maximized when both objects have the same temperature.


24-3 One mole of an ideal monatomic gas is caused to go through the cycle shown in Fig. 24-22. (a) How much work is done on the gas in expanding the gas from $a$ to $c$ along path $a b c$ ? (b) What is the change in internal energy and entropy in going from $b$ to $c$ ? (c) What is the change in internal energy and entropy in going through one complete cycle? Express all answers in terms of the pressure $p_{0}$ and volume $V_{0}$ at point $a$ in the diagram.

24-4 One mole of an ideal diatomic gas is caused to pass through the cycle shown on the PV diagram in Fig. 24-23 where $V_{2}=3 V_{1}$. Determine, in terms of $p_{1}, V_{1}, T_{1}$, and $T$ : (a) $p_{2}, p_{3}$, and $T_{3}$; and (b) $W, Q, \Delta E_{\text {int }}$, and $\Delta S$ for all three processes.

24-5 One mole of a monatomic ideal gas is taken from an initial state of pressure $p_{0}$ and volume $V_{0}$ to a final state of pressure $2 p_{0}$ and volume $2 V_{0}$ by two different processes. (I) It expands isothermally until its volume is doubled, and then its pressure is increased at constant volume to the final state. (II) It is compressed isothermally until its pressure is doubled, and then its volume is increased at constant pressure to the final state. (a) Show the path of each process on a PV diagram. For each process calculate in terms of $p_{0}$ and $V_{0}:$ (b) the heat absorbed by the gas in each part of the process; (c) the work done by the gas in each part of the process; (d) the change in internal energy of the gas, and (e) the change in entropy of the gas.

24-8 Two moles of a monatomic ideal gas are caused to go through the cycle shown in Fig. 24-24. Process $b c$ is a reversible adiabatic expansion. Also, $p_{b}=10 \mathrm{~atm}, V_{b}=1 \mathrm{~m}^{3}$ and $V_{c}=10 \mathrm{~m}^{3}$. Calculate (a) the heat added to the gas, (b) the heat leaving the gas, (c) the net work done by the gas, and (d) the efficiency of the cycle.

24-9 One mole of a monatomic ideal gas initially at a volume of 10 L and a temperature 300 K is heated at constant volume to a temperature of 600 K , allowed to expand isothermally to its initial pressure, and finally compressed isobarically to its original volume, pressure, and temperature. (a) Compute the heat input to the system during one cycle. (b) What is the net work done by the gas during one cycle? (c) What is the efficiency of this cycle?

