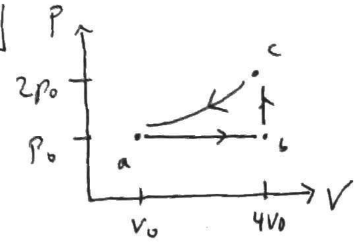
# 5 - Entropy (II)

**24-3**




(a) $W_{a\to b\to c} = W_{ab} + \underbrace{W_{bc}}_{0} = W_{ab} = p\,\Delta V = \boxed{3 p_0 V_0}$

(b) $T_b = \dfrac{p_b V_b}{\underset{1}{n} R} = \dfrac{1}{R \cdot \text{mol}}(4p_0V_0)$

$T_c = \dfrac{p_c V_c}{R} = \dfrac{1}{R\cdot\text{mol}}(8 p_0 V_0)$

$\Delta E_{bc} = \frac{d}{2} nR(T_c - T_b)$

$= \frac{3}{2}(8p_0V_0 - 4p_0V_0)$

$\boxed{\Delta E = 6 p_0 V_0}$ $(>0,\ T_c > T_b)$

$\Delta S = \int \frac{dQ}{T}$, $\quad dQ = n C_V\, dT = \frac{d}{2}\, \underset{=1}{n} R\, dT$

Const. vol process, $(C_V = \frac{d}{2} R)$

$\Rightarrow \Delta S = \int_{T_b}^{T_c} \left(\frac{d}{2} nR\right)\frac{dT}{T} = \cancel{\frac{d}{2}}\ \frac{3}{2}\cdot(8.314\ J/K)\ \ln T_c/T_b$

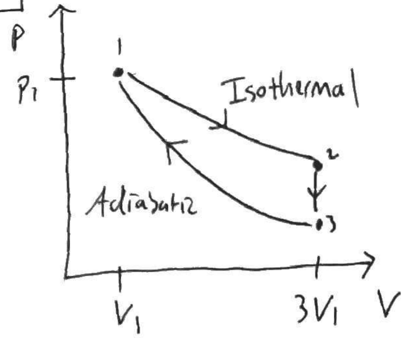
$\boxed{\Delta S = \frac{3}{2}(8.314\ J/K)\ \ln 2}$

(c) $\boxed{\begin{array}{l}\Delta E_{cycle} = 0 \\ \Delta S_{cycle} = 0\end{array}}$ because $E, S$ are <u>state variables</u> (functions),

so $\Delta E_{cyc} = E_a - E_a = 0$

$\Delta S_{cyc} = S_a - S_a = 0$.

24-4




(a) $T_2 = T_1$, $\cancel{P_2 = P_1}$ T

$\Rightarrow P_2 V_2 = P_1 V_1 \Rightarrow P_2 = \frac{1}{3} P_1$

Adiabatic: $pV^\gamma$ const.

$\Rightarrow P_1 V_1^\gamma = P_3 V_3^\gamma$

$\hookrightarrow P_3 = P_1 \cdot (V_1/V_3)^\gamma = P_1 \cdot (3^{-\gamma})$

$$\frac{T_3}{T_1} = \frac{P_3 V_3}{P_1 V_1} = (P_3/P_1)(V_3/V_1) = 3^{-\gamma} \cdot 3 = 3^{1-\gamma}$$

$\hookrightarrow \boxed{T_3 = 3^{1-\gamma} T_1}$

(Diatomic
$\hookrightarrow d = 5 \rightarrow \cancel{\gamma = 7/5}$
$\Rightarrow \gamma = 7/5 = 1.4$)

(b)

$\cancel{T_3 =}$

| | $W$ | $Q$ | $\Delta E_{int} = Q - W$ | $\Delta S = \int \frac{dQ}{T}$ |
|---|---|---|---|---|
| $1 \to 2$ | $nRT_1 \ln 3$ | $nRT_1 \ln 3$ | 0 | $nR \ln 3$ |
| $2 \to 3$ | 0 | $C_v n(\Delta T)$ | $C_v n(\Delta T)$ | $nC_v (\ln 3)\cdot(1-\gamma)$ |
| $3 \to 1$ | (Eqn. sheet) | 0 | $-W$ | 0 |

$\leftarrow$ T const, so $\Delta S = \frac{\Delta Q}{T}$

$dQ = nC_v dT$

$\Rightarrow \cancel{\Delta S} \; nC_v \ln(T_3/T_1)$

$= nC_v \ln(3^{1-\gamma})$

$= nC_v (1-\gamma) \ln 3$

$< 0$

(Since $Q < 0$) ✓

$dQ = 0$

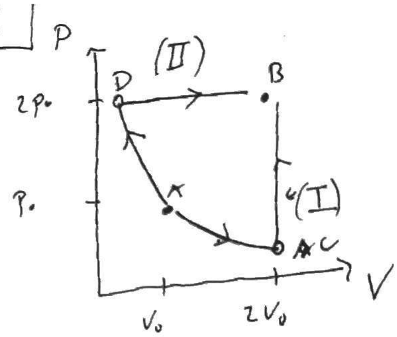
$\Delta T = T_3 - T_2 = T_1 (3^{1-\gamma}) - T_1$

$= T_1 (3^{1-\gamma} - 1)$

$< 0$

24-5

(a)




$T_C = T_A$ , $\quad T_D = T_A \qquad T_B = 4\,T_A$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \uparrow \frac{(2V_0)(2p_0)}{V_0 p_0}$

$V_C = 2V_0 \qquad p_D = 2 p_0$

$\Rightarrow p_C = \frac{1}{2} V_0 \qquad \Rightarrow V_D = \frac{1}{2} V_0$

(b) Isotherm: $Q = W = nRT \ln V_f/V_0 = p_0 V_0 \ln V_f/V_0$

(b) +c) I: $Q = W = p_0 V_0 \ln 2 \qquad (A \to C)$

II: $Q = W = -p_0 V_0 \ln 2 \;(<0) \quad (A \to D)$

Other

I: Isochoric $\to W = 0$, $Q = C_V n \Delta T = \frac{3}{2} nR\,(3T_A) = \frac{9}{2} p_0 V_0$

$\Delta V = 2V_0 - \frac{1}{2}V_0 = \frac{3}{2}V_0 \;\to$ II: Isobar $\to W = 2p_0 \cdot (\frac{3}{2}V_0)$, $Q = C_p n \Delta T = \frac{5}{2} nR \cdot (3T_A) = \frac{15}{2} p_0 V_0$

(c) $\hookrightarrow$ I: $W_{net} = p_0 V_0 \ln 2$, $\quad Q_{net} = p_0 V_0 (\frac{9}{2} + \ln 2)$

II: $W_{net} = p_0 V_0 (3 - \ln 2)$, $\quad Q_{net} = p_0 V_0 (\frac{15}{2} - \ln 2)$

(d) I: $\Delta E = Q - W = p_0 V_0 (\frac{9}{2} + \ln 2 - \ln 2) = \frac{9}{2} p_0 V_0$

II: $\Delta E = Q - W = p_0 V_0 (\frac{15}{2} - \ln 2 - 3 + \ln 2) = \frac{9}{2} p_0 V_0$ ✓

(Also, $\Delta E = \frac{3}{2} nR\,\Delta T = \frac{3}{2} nR\, 3T_a = \frac{9}{2} p_0 V_0$) ✓

(e) I: $\Delta S = \frac{Q_{iso}}{T} + \int \frac{n C_V\, dT}{T} = \frac{1}{T_a}(p_0 V_0 \ln 2) + n C_V \ln 4$

II: $\Delta S = \frac{Q_{iso}}{T} + \int \frac{n C_p\, dT}{T} = \frac{1}{T_a}(p_0 V_0 \ln 2) + n C_p \ln 4$

$\Delta S_I = nR \ln 2 + 2n C_V \ln 2 = (R + 2C_V) \ln 2 \quad$ (n=1 mol)

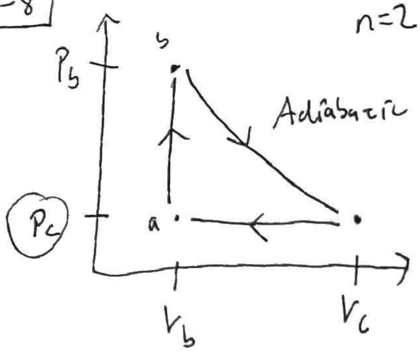
$\Delta S_{II} = -nR \ln 2 + 2n C_p \ln 2 = (2C_p - R) \ln 2$

agree since $C_p = C_V + R$

$\hookrightarrow 2C_p - R = 2C_V + R$ ✓

(Monatomic: $C_V = \frac{3}{2} R$)

24-8




$n=2$, $P_b = 10$ atm, $V_b = 1 m^3$, $V_c = 10 m^3$

(a) $Q = Q_{ab} + Q_{bc} + Q_{ca}$

$= 2\, C_V (T_b - T_a) + 0 + 2\, C_p (T_a - T_c)$

$= 2\, C_V (T_b - T_a) + 2\, C_p (T_a - T_c)$

Adiabatic $\rightarrow$ $pV^\gamma = \text{const.}$ Need to get, $P, T$.

$\Rightarrow P_c V_c^\gamma = P_b V_b^\gamma$

$P_c = P_b \cdot (V_b/V_c)^\gamma = P_b \cdot 10^{-\gamma}$

(Monatomic: $\gamma = \frac{3+2}{3} = 5/3$)

$T_b = \frac{P_b V_b}{2R}$, $T_c = \frac{P_c V_c}{2R} = T_b \cdot 10^{1-\gamma} < T_b$

$\left( = \frac{10^6 J}{16\, J/K} = 63000 K! \right)$

$T_a = \frac{P_a V_a}{2R} = T_b \cdot 10^{-\gamma} < T_b$

$C_V = \frac{d}{2} R = 3/2 R$

$C_p = (1 + d/2) R = 5/2 R$

$$\Rightarrow \boxed{\begin{aligned} Q_{net} &= 2\, C_V T_b (1 - 10^{-\gamma}) + 2\, C_p T_b (10^{-\gamma} - 10^{1-\gamma}) \\ &= 3 R T_b \underbrace{(1 - 10^{-\gamma})}_{0.98 \sim 1} + 5 R T_b \underbrace{(10^{-\gamma} - 10^{1-\gamma})}_{-0.19 \sim -1/5} \\ &\approx 3 R T_b - R T_b = 2 R T_b = 2 P_b V_b \end{aligned}}$$

$= 2 \cdot 10^6$ J

(b) Oh sorry: $Q_{added} = Q_{ab} = 3 R T_b (1 - 10^{-\gamma}) \approx 3 \cdot 10^6$ J

$Q_{leaving} = Q_{ca} = +5 R T_b (10^{-\gamma} - 10^{1-\gamma}) \approx -1 \cdot 10^6$ J

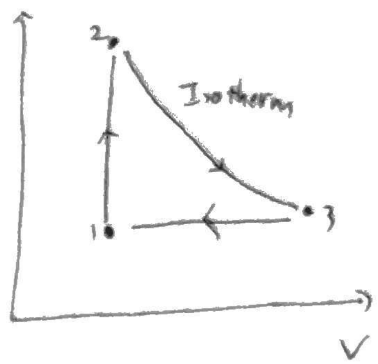
(c) $W_{net} = Q_{net} \approx 2 \cdot 10^6$ J $\leftarrow \Delta E_{net} = 0 = Q_{net} - W_{net}$

(d)

$\varepsilon = \frac{W_{net}}{Q_{added}} = 1 - \frac{|Q_{leaving}|}{Q_{added}} = \cancel{1 + \frac{3(1-10^{-\gamma})}{}}$

$= 1 - \frac{5 (10^{1-\gamma} - 10^{-\gamma})}{3 (1 - 10^{-\gamma})} \approx 1 - \frac{2}{3} = \boxed{1/3}$

24-4




(a) $Q_{in} = Q_{12} + Q_{23}$

$Q_{12} = W_{12} + \Delta E_{12}$, $W_{12} = 0$, $\Delta E_{12} > 0$

$Q_{23} = W_{23} + \Delta E_{23}$, $W_{23} > 0$, $\Delta E_{23} = 0$

$$\left( Q_{out} = \underset{nC_p \Delta T < 0}{Q_{31}} \right)$$

$n = 1$

$d = 3$

$P_2 = P_1 \cdot \frac{T_2}{T_1}$

$P_3 = P_1$

$\Rightarrow P_2/P_3 = T_2/T_1$ $\longrightarrow$

$$Q_{in} = n C_V \Delta T + W_{23}$$

$$= \frac{3}{2} n R \Delta T + n R T_2 \ln(V_3/V_2)$$

$$= \frac{3}{2} n R (T_2 - T_1) + n R T_2 \ln(P_2/P_3)$$

$$= \frac{3}{2} n R (T_2 - T_1) + n R T_2 \ln(T_2/T_1)$$

$$= R \cdot \left[\frac{3}{2}(300\text{ K}) + 600\text{ K} \ln 2\right]$$

$$= R \cdot 300\text{ K} \underbrace{\left[3/2 + 2 \ln 2\right]}_{(1.5 + 1.4)} = \boxed{7200\text{ J}}$$

(b)

$$W_{net} = Q_{net} = Q_{in} + Q_{out}$$

$$-Q_{out} = n C_p \Delta T = n \frac{5}{2} R (T_1 - T_3) = n \frac{5}{2} R (T_2 - T_2) = \frac{5}{2} \cdot R \cdot 300\text{K} =$$

$$\Rightarrow Q_{net} = -nR(T_2 - T_1) + nR T_2 \ln 2$$

$$= R \cdot 300\text{K} \cdot [-1 + 2\ln 2] = \boxed{963\text{ J} = W_{net}}$$

(c)

$$\varepsilon = \frac{W_{net}}{Q_{in}} = \frac{963}{7200} = \boxed{13.4\%}$$