5 - Entropy

24-16

$$\varepsilon = 1 - \frac{T_L}{T_H} \hookrightarrow \text{~~Want to compare~~ } \cancel{\frac{T_L + \Delta}{T_H} \text{ vs. } \frac{T_L}{T_H+\Delta}}$$

$$= \frac{(T_H - T_L)}{T_H} \qquad \cancel{\frac{T_L+\Delta}{T_H} - \frac{T_L}{T_H+\Delta} = \frac{1}{T_H}}$$

If we do $T_H \to T_H + \Delta$, or $T_L \to T_L - \Delta$, then in either case the numerator is increased: $(T_H - T_L) + \Delta$ by same amt.

However, $T_H \to T_H + \Delta$ also increases the <u>denominator</u>.

So we would rather do $\boxed{T_L \to T_L - \Delta}$

24-17

$$\begin{tikzcd} \boxed{T_H} \arrow[r] & E_1 \arrow[r] \arrow[d] & \boxed{T_M} \arrow[r] & E_2 \arrow[r] \arrow[d] & \boxed{T_L} & \cong & T_H \arrow[r] & E_{12} \arrow[r] \arrow[d] & T_L \\ & W_1 & & W_2 & & & & W_1 + W_2 & \end{tikzcd}$$

We can combine the two engines into one "super-engine" directly connecting the hottest and lowest points, T_H & T_L.

This will always be <u>no more efficient</u> than a Carnot engine b/w T_H, T_L.

(However, in practice sometimes this lumped design is better e.g. <u>cheaper</u>.)

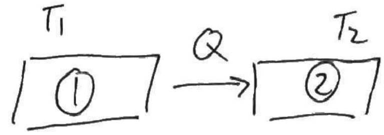
24-21

Put something (a sandwich) in the fridge, and regard just "the sandwich" as "the system".

$\Delta S_{\text{sandwich}} < 0$, since it loses Heat.

But $\Delta S_{\text{sandwich}} + \Delta S_{\text{fridge}} + \Delta S_{\substack{\text{rest of} \\ \text{the universe}}} > 0$

24-8



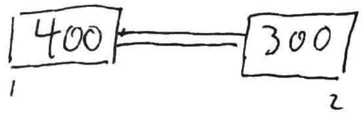
$$\Delta S = \Delta S_1 + \Delta S_2$$
$$= \frac{Q}{T_2} - \frac{Q}{T_1} = Q\left(\frac{T_1 - T_2}{T_1 T_2}\right)$$

If we hold T_1 fixed, vary T_2: $\Delta S = \frac{Q}{T_1} \cdot \frac{\Delta T}{T_2} = \frac{200\,J}{400\,K} \cdot \frac{\Delta T}{T_2} = \frac{1}{2} \frac{\Delta T}{T_2}\ J/K$

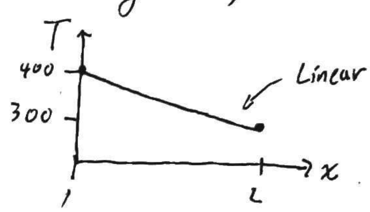
~~(a)~~ $\hookrightarrow \frac{\Delta T}{T_2} = (T_1/T_2) - 1$

As $T_2 \to T_1$, the net entropy change goes $\to 0$.

24-9



(a) In steady state,



(b+c) When heat flows through the rod, in steady state, there is no <u>net Q into the rod</u>:

i.e. $Q_{out} = Q_{in} = 1200\,J$

$$\boxed{\Delta S_{rod} \simeq \frac{Q_{net}}{T} = 0}$$

So $\Delta S = \Delta S_1 + \Delta S_2$

$$= -\frac{1200}{400} + \frac{1200}{300} = -3 + 4 = \boxed{1\ J/K}$$

24-1

$$dQ = C_V\, dT = \underset{\#\text{ mols}}{n} \cdot A\, T^3\, dT$$

$$\Delta S = \int \frac{dQ}{T} = \int_{T_1}^{T_2} \underbrace{n \cdot A \frac{T^3}{T}}_{= nAT^2} dT = \boxed{\frac{An}{3}(T_2^3 - T_1^3)}$$

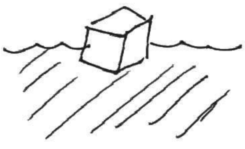
Aluminum:

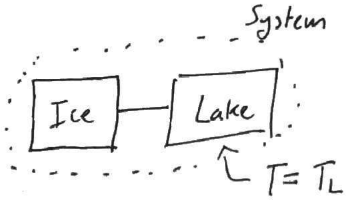
$A \cdot n = 3.15 \cdot 10^{-5}\ J/mol\,K \cdot 4.8\ mol \doteq 3.5 \cdot 10^{-5}\ J/K = 15 \cdot 10^{-5}\ J/K$

$T_2^3 - T_1^3 = 10^3 - 5^3 = 1000 - 125 \approx 900$

$\hookrightarrow \Delta S = \frac{An}{3}(T_2^3 - T_1^3) = \frac{15}{3} \cdot 10^{-5} \cdot 900 = 5 \cdot 900 \cdot 10^{-5} = 4500 \cdot 10^{-5} = \boxed{0.045\ J/K}$

24 - 6





$T = T_L = \text{const.}$ (lake much larger than ice cube)

Three stages: • Ice warms to $0°C$ $\rightarrow$ Heat $Q = c_{ice}\, m\, \Delta T$, $dQ = c_{ice}\, m\, dT$

• Ice melts $\rightarrow$ latent heat $Q = L_f m$

• Melted ice warms to $15°C$ $\rightarrow$ $dQ = c_{water}\, m\, dT$

$$\Delta S_{ice} = \int_{-10°C}^{0°C} \frac{dQ}{T} + \frac{L_f m}{273} + \int_{0°C}^{15°C} \frac{dQ}{T}$$

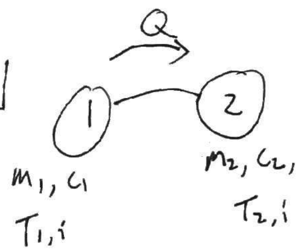
$$= \int_{+263}^{273} \frac{c_{ice}\, m\, dT}{T} + \frac{L_f m}{273} + \int_{273}^{285} \frac{c_{water}\, m\, dT}{T}$$

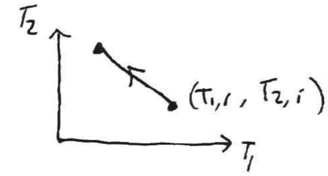
$$= c_{ice}\, m \ln\frac{273}{263} + \frac{L_f m}{273} + c_{water}\, m \ln\frac{285}{273}$$

$$\Delta S_{lake} = -\left(\frac{c_{ice}\, m\, \Delta T}{T_L} + \frac{L_f m}{T_L} + \frac{c_{water}\, m\, \Delta T}{T_L}\right) = -\frac{m}{285}\left[c_{ice} \cdot 10\,K + L_f + c_{water} \cdot 15\,K\right]$$

$$\Delta S_{tot} = \Delta S_{ice} + \Delta S_{lake}$$

24-7





(a) $-dQ_1 = dQ_2$

$$-m_1 c_1\, dT_1 = m_2 c_2\, dT_2$$

$$\Rightarrow \frac{dT_2}{dT_1} = -\frac{m_1 c_1}{m_2 c_2} \Rightarrow \boxed{T_2(T_1) = T_{2,i} + \frac{m_1 c_1}{m_2 c_2}(T_{1,i} - T_1)}$$

(b) $dS = dS_1 + dS_2 = \cancel{dQ}$

$$= \frac{dQ_1}{T_1} + \frac{dQ_2}{T_2} = \cancel{-\frac{dQ_2}{T_1} + \frac{dQ_2}{T_2} = \left(\frac{T_1 - T_2}{T_1 T_2}\right) dQ_2}$$

$$= m_1 c_1 \frac{dT_1}{T_1} + m_2 c_2 \frac{dT_2}{T_2} \quad \hookrightarrow \left(\Delta S = mc \ln\frac{T_f}{T_i} = \int dS\right)$$

Integrating $\Rightarrow$ $$\boxed{\Delta S = \underbrace{m_1 c_1 \ln\left(\frac{T_1}{T_{1,i}}\right)}_{<0} + \underbrace{m_2 c_2 \ln\left(1 + \frac{m_1 c_1}{m_2 c_2} \cdot \frac{T_{1,i} - T_1}{T_{2,i}}\right)}_{>0}}$$

(c) Entropy is max. when

$$\frac{d}{dT_1}[\Delta S] = 0 \Leftrightarrow \frac{dS_k}{dT_1} = 0 \rightarrow \frac{dS}{dT_1} = m_1 c_1 \frac{dT_1}{T_1}\left(\frac{1}{dT_1}\right) + m_2 c_2 \frac{dT_2}{dT_1} \frac{1}{T_2}$$

$-\frac{m_1 c_1}{m_2 c_2}$

$$= \frac{m_1 c_1}{T_1} + \frac{m_2 c_2}{T_2}\left(-\frac{m_1 c_1}{m_2 c_2}\right) = m_1 c_1\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$dS/dT_1 = 0 \Leftrightarrow 0 = \frac{1}{T_1} - \frac{1}{T_2}$$

$$\boxed{T_1 = T_2}$$