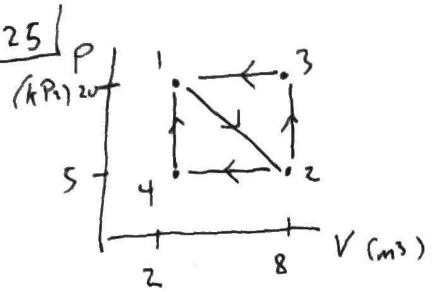
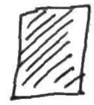
3 Work and Internal Energy (PV diagrams)

23.25

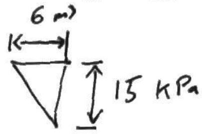


Path 1:

$W = \int p\,dV = W_{12} + W_{23} + W_{31}$

$=$  $+ 0 + -1 \times$ 

$= -$  $= -\frac{1}{2}(6\ m^3)(15\ kPa) = \boxed{-45\ kJ}$

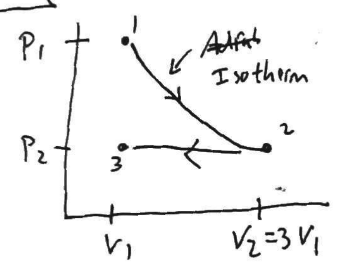


Path 2:

$W = \int p\,dV = W_{12} + W_{24} + W_{41}$

$=$  $+ 0 = +$  $= \frac{1}{2}(6\ m^3)(15\ kPa) = \boxed{45\ kJ}$

23.26



(a) Isotherm $\Rightarrow$ T const. $\Rightarrow pV$ const

$\Rightarrow p_1 V_1 = p_2 V_2 = p_2(3V_1)$

$\Rightarrow p_2 = p_1/3$

$pV = \text{const} \Rightarrow p(V) = \frac{(p_1 V_1)}{V}$ $\quad \left(p(V_1) = \frac{p_1 V_1}{V_1} = p_1\ \checkmark\right)$

So ~~XXXXXXXXXXXX~~

$(V_1, p_1) \to (3V_1, p_1/3) \to (V_1, p_1/3)$

Work done by gas

$W_{1\to3} = \cancel{\int p\,dV =} W_{1\to2} + W_{2\to3}$

$= \int_{V_1}^{3V_1} p(V)\,dV + \int_{3V_1}^{V_1} p(V)\,dV$

$= \int_{V_1}^{3V_1} \frac{p_1 V_1}{V}\,dV + \int_{3V_1}^{V_1} (p_1/3)\,dV$

$= p_1 V_1 [\log V]_{V_1}^{3V_1} + (p_1/3)\cdot(\cancel{2}\,V_1 - 3V_1)$

$= p_1 V_1 \log 3 - \frac{2}{3} p_1 V_1 = p_1 V_1 [\log 3 - 2/3] \sim 1.1\, p_1 V_1$

(Work done on gas $= -W_{1\to3} = \boxed{-p_1 V_1 [\log 3 - 2/3]}$)

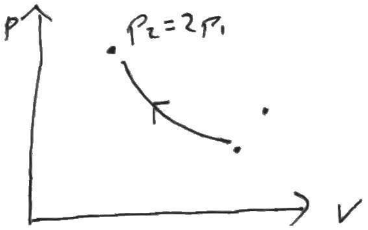
(b) $E_{int} = \frac{d}{2} NkT = \frac{d}{2} pV$ (d ← # d.o.f.)

$\Rightarrow \Delta E_{int} = \frac{d}{2}(p_3 V_3 - p_1 V_1) = \frac{d}{2}((p_1/3)V_1 - p_1 V_1) = \frac{d}{2}(-\frac{2}{3} p_1 V_1)$

$= \boxed{-\frac{d}{3} p_1 V_1}$

23.30

$T_1 P_1 \longrightarrow \boxed{200\,W} \longrightarrow T_2 P_2 \quad \leftarrow p_2 = 2p_1$



Adiabatic: $pV^\gamma = \text{const.}$

Air: $\gamma \simeq 1.4$

(a) $p_1 V_1^\gamma = p_2 V_2^\gamma = 2p_1 V_2^\gamma$

$\rightarrow V_1^\gamma = 2V_2^\gamma \rightarrow V_2^\gamma = \frac{1}{2} V_1^\gamma$

$\Rightarrow V_2 = V_1 \cdot (\tfrac{1}{2})^{1/\gamma} = V_1 \cdot 2^{-1/\gamma}$ $\quad (2^{-1/1.4} \simeq 0.61)$

(N const.): $T_2 = T_1 \cdot \frac{p_2}{p_1} \cdot \frac{V_2}{V_1} = T_1 \cdot (2) \cdot (2)^{-1/\gamma}$

$= T_1 \cdot 2^{1-1/\gamma} = \boxed{T_1 \cdot 2^{\frac{\gamma-1}{\gamma}}}$ $\quad (2^{0.4/1.4} = 1.2)$

(b) In 1 second, work done on the gas $= 200\,W \cdot 1s = 200\,J$

For convenience, I'll just use the equation

$W = \frac{1}{\gamma-1}(p_2 V_2 - p_1 V_1)$ $\leftarrow$ (Careful about overall sign here)

$= \frac{1}{\gamma-1}(2p_1 2^{-1/\gamma} V_1 - p_1 V_1) = \boxed{\frac{p_1 V_1}{\gamma-1}\left(2^{\frac{\gamma-1}{\gamma}} - 1\right)}$

$\Rightarrow \boxed{V_1 = \frac{W(\gamma-1)}{p_1} \frac{1}{2^{\frac{\gamma-1}{\gamma}} - 1} = \frac{200\,J}{p_1} \cdot \frac{\gamma-1}{2^{\frac{\gamma-1}{\gamma}} - 1}}$

If $\gamma = 1.4 \Rightarrow \frac{1.4-1}{2^{0.4/1.4} - 1} = 6.4$

$p_1 \approx 1\,atm = 10^5\,Pa \Rightarrow V_1 = \frac{200 \cdot 6.4}{10^5}\,m^3 = 0.0128\,m^3 = 12.8\,L$

23.14

vdW: $\left(p + \frac{an^2}{V^2}\right)(V - nb) = nRT$

(Note: This replaces the ideal gas law, so we can't use $pV = nRT$) in this problem

Isothermal expansion $\Rightarrow$ T constant

To calculate $W = \int p\,dV$, need to find $p(V)$. So solve for p:

$$\left(p + \frac{an^2}{V^2}\right)(V-nb) = nRT$$

$$p + \frac{an^2}{V^2} = \frac{nRT}{V-nb}$$

$$p = \frac{nRT}{V-nb} - \frac{an^2}{V^2}$$

Now integrate this:

$$W = \int_{V_i}^{V_f} p\,dV = \int_{V_i}^{V_f}\left(\frac{nRT}{V-nb} - \frac{an^2}{V^2}\right)dV$$

$$= \left[nRT \cdot \ln(V-nb) + \frac{an^2}{V}\right]_{V_i}^{V_f}$$

$$\left(\frac{d}{dV}\ln(V-nb) = \frac{1}{V-nb} \qquad \frac{d}{dV}\frac{1}{V} = -\frac{1}{V^2}\right)$$

$$\boxed{W = nRT \cdot \ln\frac{V_f - nb}{V_i - nb} + an^2\left(\frac{1}{V_f} - \frac{1}{V_i}\right)}$$

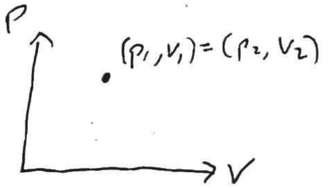
23.16

$V_1 = V_2 = V$

$p_1 = p_2 = p_0$

$T_1 \neq T_2$

$\Rightarrow$



Same point on pV diagram!

Using $E_{int} = NkT \cdot \frac{d}{2} = \frac{d}{2} pV = \frac{5}{2} pV$ (ideal gas; diatomic)

Since $p_1 V_1 = p_2 V_2 = p_0 V$, then

$$\boxed{E_1 = \tfrac{5}{2} p_1 V_1 = \tfrac{5}{2} p_0 V = \tfrac{5}{2} p_2 V_2 = E_2}$$

What's happened? N has changed $(N_1/N_2 = T_2/T_1)$

Since the room is leaky, some of the hot air escaped outside, until the pressures equalized.

So the total E_{int} (inside) hasn't changed, although it does still feel warmer (since $T_2 > T_1$)

23.XX ← Written by Dan Parker (I think)

(a) On PV diagram, so want in terms of P, V:

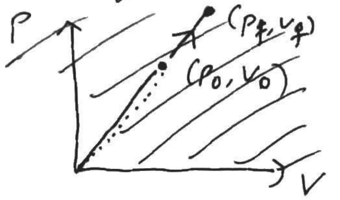
$$T/V^2 = \left(\frac{T}{V}\right)\cdot\frac{1}{V} = \frac{1}{Nk}\cdot\frac{P}{V}$$

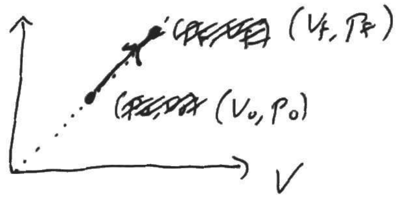
$$PV = NkT \Rightarrow \frac{T}{V} = \frac{P}{Nk}$$

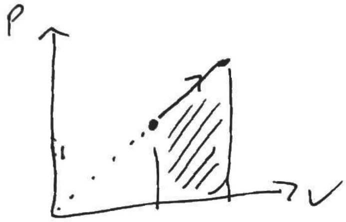
So $T/V^2 = \text{const} \Leftrightarrow P/V = \text{const} \equiv A$

$$\Rightarrow P = A\cdot V = (P_0/V_0)\cdot V$$

This a straight line through the origin:





(b) 

$$W = \int p\,dV \neq$$ 

$$= \int_{V_0}^{V_f} (P_0/V_0)\cdot V\,dV$$

$$= \boxed{\frac{1}{2}(P_0/V_0)(V_f^2 - V_0^2)}$$