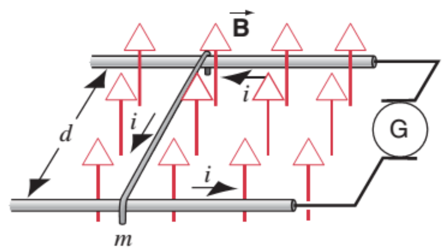
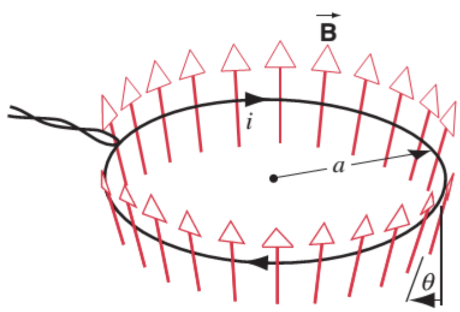
16 Magnetic force on wires and dipoles

Wire:	$\mathrm{d}\vec{F} = I\,\mathrm{d}\vec{l} \times \vec{B}$	$\tau = N_{loops} I A \hat{n} \times B$	$\vec{\mu} = I A \hat{n}$
Dipole:	$\vec{\tau} = \vec{\mu} \times \vec{B}$	$U = -\vec{\mu} \cdot \vec{B}$	$F_z = -\frac{\mathrm{d}U}{\mathrm{d}z} = \mu_z \frac{\mathrm{d}B}{\mathrm{d}z}$

1. *(RHK Exercise 32.32)* A metal wire of mass m slides without friction on two horizontal rails spaced a distance c apart, as shown below. The track lies in a vertical uniform magnetic field $\vec{B}$. A constant current I flows from the generator along one rail, across the wire, and back down the other rail. Find the velocity of the wire as a function of time, assuming it to be at rest at $t = 0$.



2. *(RHK Problem 32.18)* The figure below shows a wire ring of radius a at right angles to the general direction of a radially symmetric diverging magnetic field. The magnetic field at the ring is everywhere of the same magnitude B, and its direction at the ring is everywhere at an angle θ with a normal to the plane of the ring. Find the magnitude and direction of the force the field exerts on the ring if the ring carries a current I as shown. *(The twisted wire leads have no effect on this problem.)*



3. *(RHK Exercise 35.11)* A charge q is uniformly distributed around a thin ring of radius r. The ring is rotating about an axis through its center and perpendicular to its plane at angular frequency ω. *(a)* Show that the magnetic moment due to the rotating charge is $\mu = (1/2)q\omega r^2$. *(b)* If L is the angular momentum of the ring, show that $\mu/L = q/2m$.

4. *(RHK Problem 35.1)* A thin, plastic disk of radius R has a charge q uniformly distributed over its surface. If the disk rotates at an angular frequency ω about its axis, show that magnetic dipole moment of the disk is

$$\mu = \frac{\omega q R^2}{2} \tag{16.1}$$

5. A circular loop of wire with radius r and mass m is centered on the z-axis. A current I flows counterclockwise through the wire. An external, *non-uniform* magnetic field $\vec{B} = B_0 z^2 \hat{z}$ is applied. *(a)* What is the magnetic moment of the loop? *(b)* What is the potential energy of the loop, as a function of its height along the z-axis? *(c)* What is the corresponding force on the loop? *(d)* If the loop intially is stationary at height h_0, find its height $h(t)$ as a function of time.