15 Magnetic force on particles

Particle:	$\vec{F} = q\vec{v} \times \vec{B}$	$F = qvB\sin\theta$	$a_c = \dfrac{v^2}{r}$
Dipole:	$\vec{\tau} = \vec{\mu} \times \vec{B}$	$U = -\vec{\mu} \cdot \vec{B}$	$F_z = -\dfrac{\mathrm{d}U}{\mathrm{d}z} = \mu_z \dfrac{\mathrm{d}B}{\mathrm{d}z}$

1. *(Cyclotron motion)* A particle with charge $+q$ and mass m is placed in a uniform magnetic field $\vec{B} = B\hat{z}$. At $t = 0$, the particle is at $x = y = z = 0$ and has velocity $\vec{v} = v\hat{x}$. At a later time, the particle may have more general velocity $\vec{v}(t) = (v_x(t), v_y(t), v_z(t))$.

(a) What is the force on the particle at $t = 0$? What is it at a later time t, in terms of B and the components of $\vec{v}(t)$?

(b) Use Newton's 3rd law to write down equations relating $\vec{v}(t)$ and $\vec{a}(t) = \mathrm{d}\vec{v}/\,\mathrm{d}t$ at any time t.

(c) Explain why $|v(t)|$ remains constant, and why $v_z(t) = 0$.

(d) Solve the equations you found in part b to find $v_x(t)$ and $v_y(t)$.

(e) Integrate to find $x(t), y(t), z(t)$.

(f) You should get uniform circular motion. What are the radius r and angular frequency ω for this motion, as a function of q, m, B?

2. *(RHK Problem 32-6)* The diagram below shows a design for a *mass spectrometer*, a device for measuring the masses of ions (charged particles). An ion of mass m and charge $+q$ is produced at rest in the source S, and then accelerated by a potential difference ΔV into the entry slit of a magnetic chamber. The ion then enters a uniform magnetic field $\vec{B}$ and moves in a semicircle until it strikes a photographic plate at a distance x from the entry slit. Show that the ion mass m is given by

$$x = \frac{B^2 q}{8\Delta v} x^2 \qquad (15.1)$$