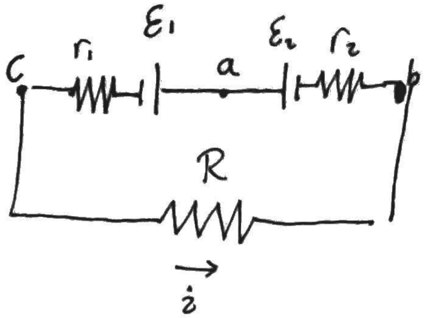
12 DC Circuits – Resistors and Batteries

1]



(a) i is the same everywhere in the circuit, because there are no branches.

(All junctions are like $\xrightarrow{i_{in}} \bullet \xrightarrow{i_{out}}$ ⇒ Apply 1[st] Law)

Loop rule:

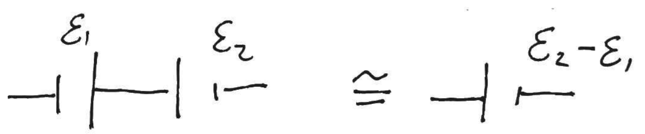
Batteries: $-\mathcal{E}_1$, $+\mathcal{E}_2$

Resistors: $-i r_2$, $-iR$, $-i r_1$

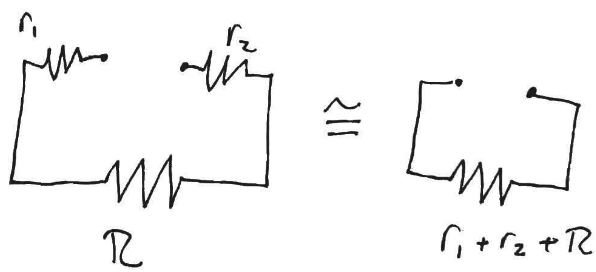
$$-\mathcal{E}_1 + \mathcal{E}_2 - i(r_2 + R + r_1) = 0$$

Solve for I: $$\boxed{i = \frac{\mathcal{E}_2 - \mathcal{E}_1}{r_1 + r_2 + R}}$$

Alternatively:



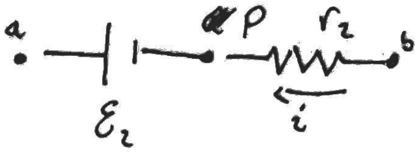
(Batteries in series)



(Resistors in series)

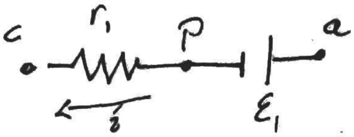
$$\Rightarrow \quad i = \frac{\mathcal{E}_{eff}}{R_{eff}} = \frac{\mathcal{E}_2 - \mathcal{E}_1}{r_1 + r_2 + R}$$

(b) Between points a and b:



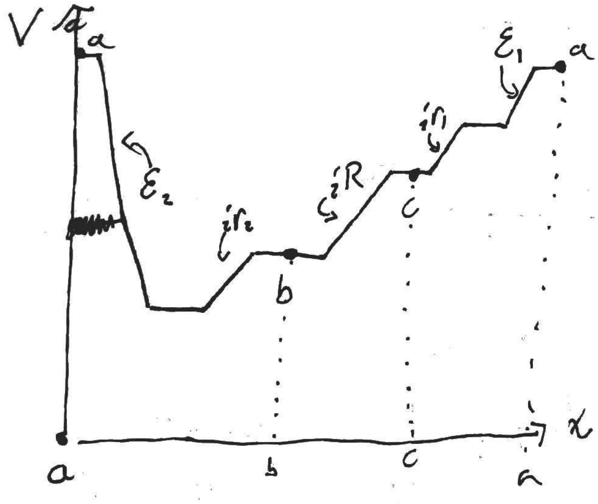
$$V_a - V_p = \mathcal{E}_2$$
$$V_b - V_p = i r_2$$
$$\Rightarrow V_a - V_b = \mathcal{E}_2 - i r_2$$

(c) Between points a and c:

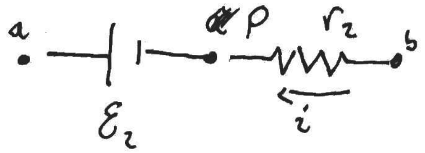


$$V_a - V_p = \mathcal{E}_1$$
$$V_p - V_c = i r_1$$
$$\Rightarrow V_a - V_c = \mathcal{E}_1 + i r_1$$

(d) (Assuming $\mathcal{E}_2 - \mathcal{E}_1 > 0$, so $i > 0$ - that is, picked correct direction for i)



(b) Between points a and b:



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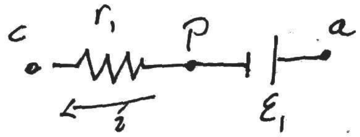

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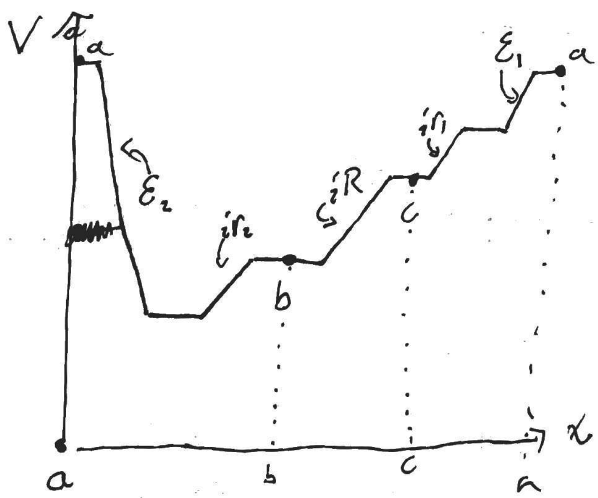
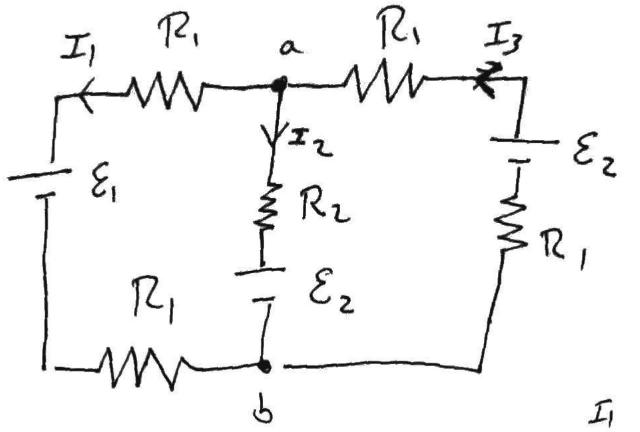


2]

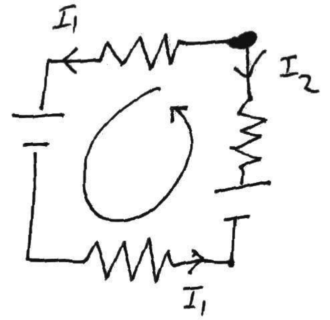


(a) In this problem, we'll use Kirchoff's rules.

Junction rule: $I_3 = I_1 + I_2$

Loop rule (2 of them):

$\mathcal{E}_1 = 2$
$\mathcal{E}_2 = 6$
$R_1 = 2$
$R_2 = 4$

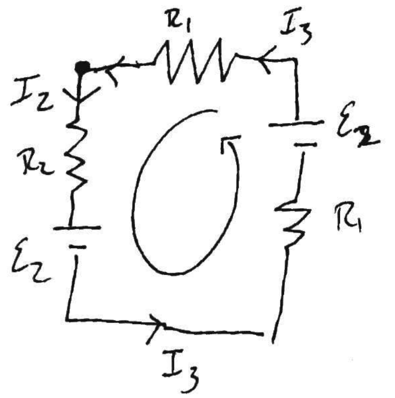


$$0 = -I_1 R_1 - \mathcal{E}_1 - I_1 R_1 + \mathcal{E}_2 + I_2 R_2$$
$$= (\mathcal{E}_2 - \mathcal{E}_1) - 2 I_1 R_1 + I_2 R_2$$
$$= 4 - 4 I_1 + 4 I_2$$

$I_3 = I_1 + I_2$

$I_1 = I_2 + 1 \Rightarrow I_3 = 2I_2 + 1$

Second loop:



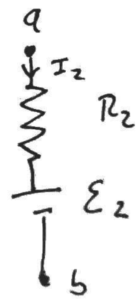
$$0 = -I_2 R_2 - \mathcal{E}_2 - I_3 R_1 + \mathcal{E}_2 - I_3 R_1$$
$$= -I_2 R_2 - 2 I_3 R_1$$
$$0 = 4 I_2 + 4 I_3$$
$$\Rightarrow I_2 = -I_3 = -2I_2 - 1$$

$\Rightarrow \boxed{I_2 = -1/3}$ A $\Rightarrow \boxed{I_1 = 1 + I_2 = +2/3}$

$\Rightarrow \boxed{I_3 = I_1 + I_2 = 1/3}$

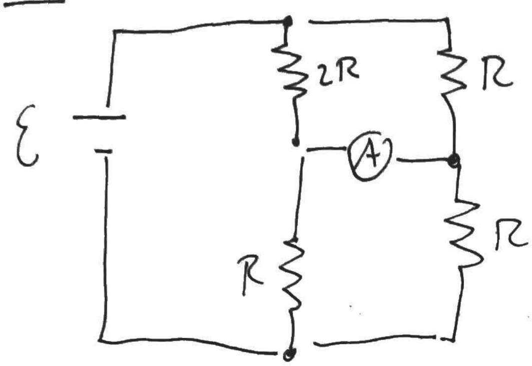
(All in amps)
if $\mathcal{E}$ ~ volts
R ~ ohms

(b)



$$V_a - V_b = I_2 R_2 + \mathcal{E}_2 = -\frac{1}{3}\cdot 4 + 6 = 14/3$$

3

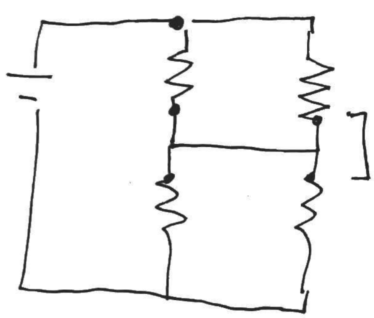


What is the current through (A)? ← 0 resistance

This problem can be solved by applying Kirchoff's laws, though it ends up being a bit tedious.

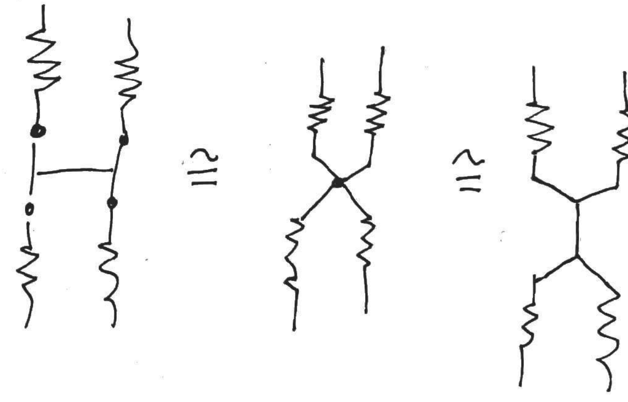
Instead, we use a trick.

Since (A) has zero resistance, replace it w/ ideal wire:



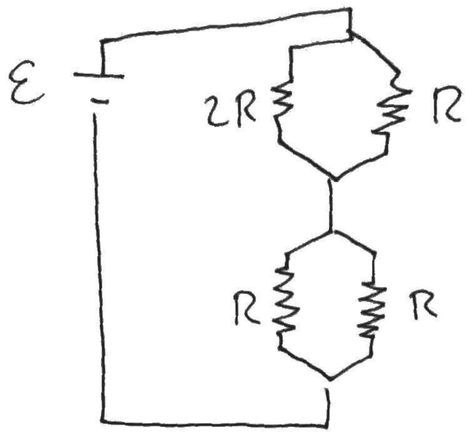
Because connected by ideal wires, everything in this area at <u>same</u> <u>potential</u>.

Can rearrange like:

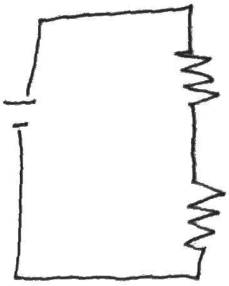


3, continued

So really the circuit is equivalent to a series/parallel:



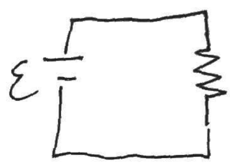
Combine Parallel $\cong$



$(R^{-1} + \frac{1}{2}R^{-1})^{-1} = \frac{2}{3}R$

$(R^{-1} + R^{-1})^{-1} = \frac{1}{2}R$

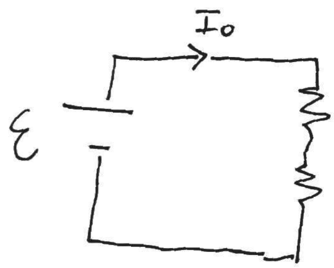
Series $\cong$



$R_{eff} = \frac{1}{2}R + \frac{2}{3}R = \frac{7}{6}R$

$\Rightarrow$ Current coming out of battery is $I_0 = \mathcal{E}/R_{eff} = \frac{6}{7}\mathcal{E}/R$

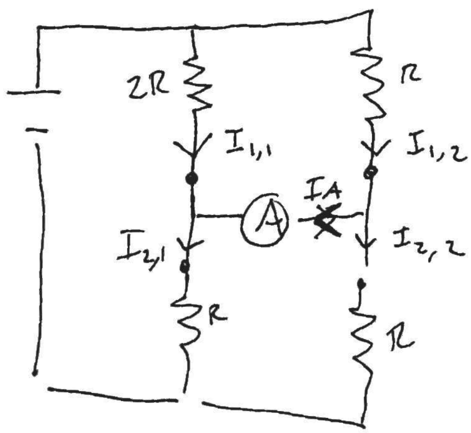
Now we work backwards:



$2/3\ R \Rightarrow V_1 = \frac{2}{3} I_0 R = \frac{4}{7}\mathcal{E}$

$1/2\ R \Rightarrow V_2 = \frac{1}{2} I_0 R = \frac{3}{7}\mathcal{E}$

With these voltage diff, find currents through each resistor.



$I_{1,1} = V_1/2R = \frac{2}{7}\frac{\mathcal{E}}{R}$

$I_{1,2} = V_1/R = 4/7\ \mathcal{E}/R$

$I_{2,1} = I_{2,2} = V_2/R = 3/7\ \mathcal{E}/R$

1st Law: /Junction rule

$I_A = I_{2,1} - I_{1,1} = \frac{3}{7}\mathcal{E}/R - \frac{2}{7}\mathcal{E}/R$

$= \boxed{1/7\ \mathcal{E}/R}$