11 Capacitors

$$Q = C\Delta V \qquad C_{\text{eq, parallel}} = \sum_i C_i \qquad C^{-1}_{\text{eq, series}} = \sum_i C_i^{-1}$$

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV \qquad u = \frac{1}{2}\epsilon_0 E^2$$

$$\text{Dielectric: } \epsilon_0 \to \kappa\epsilon_0 \qquad C \to \kappa C \qquad Q_{enc} = \int_{\mathcal{S}} (\kappa\epsilon_0)\vec{E}\cdot \mathrm{d}\vec{A}$$

11.a Calculating capacitance

1. Place charges. Sometimes this is $+Q$ on one piece ("plate") of the capacitor and charge $-Q$ on the other plate. Other times it is $+Q$ on one and 0 on the other ("grounded").
2. Calculate the electric field using Gauss' law and/or superposition.
3. Find the electric potential difference between the plates by integrating on any convenient curve, $\Delta V = -\int_-^+ \vec{E}\cdot \mathrm{d}\vec{s}$. (Sometimes you should instead find the potential difference between one of the plates and $V(r=\infty)$.)
4. The capacitance is then $C = Q/\Delta V$. Simplify your expression until it doesn't involve Q anywhere. You should get something with units of ϵ_0·[Area]/[Distance] $\sim \epsilon_0\cdot$ [Length].

1. A non-conducting spherical shell of radius R and charge $-Q$ is held in place with its center a distance h above a thin insulating sheet with area A and charge $+Q$. ($h \geq R$) *(a)* Calculate the capacitance of this system, $C = Q/\Delta V$. *(b)* Explain what happens in the limit where $A \to \infty$ and $h \to R$, when R is held fixed. *(c)* Fix R and Q, and take $A \to \infty$. Calculate the stored energy U, and $\mathrm{d}U \,/\, \mathrm{d}h$. *(d)* If the sheet is held fixed, what must be the external force (magnitude and direction) applied to keep the sphere in place?

2. *(Zangwill 5.2)* A spherical conducting shell with radius b is concentric with and encloses a conducting ball with radius a. Compute the capacitance $C = Q/\Delta V$ when *(a)* the shell is grounded and the ball has charge Q. *(b)* the ball is grounded and the shell has charge Q.

3. *(Zangwill 5.3)* A capacitor is formed from three very long, concentric, conducting cylindrical shells with radii $a < b < c$. A fine wire connects the inner (a) and outer (c) shells, and a uniform positive charge per unit length λ_b is placed on the middle (b) shell. Compute the capacitance per unit length, $C/L = \lambda_b/|V_b - V_a|$.

11.b Parallel and series

- It is often useful to consider a complicated capacitor as an *composite* of simpler capacitors, possibly infinitesimal, connected in series and/or parallel.

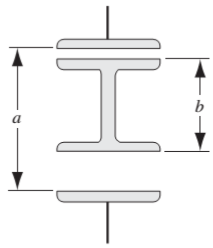

FIGURE 30-29.

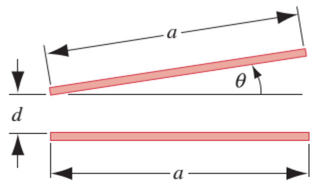

FIGURE 30-34.

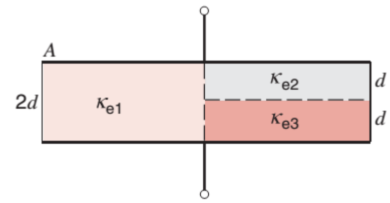

FIGURE 30-39.

4. *(RHK 30-3)* Figure 30-29 shows two capacitors in series, the rigid center section of length b being movable vertically. Show that the equivalent capacitance of the series combination is independent of the position of the center section and is given by

$$C = \epsilon_0 A/(a - b)$$

5. *(RHK 30-8)* A capacitor has square plates, each of side length a, making an angle θ with each other as shown in Fig. 30-34. Show that for small θ the capacitance is given by

$$C = \frac{\epsilon_0 a^2}{d}\left(1 - \frac{a\theta}{2d}\right)$$

(Hint: the capacitor may be divided into differential strips that are effectively parallel.)

6. *(RHK 30-20)* What is the capacitance of the capacitor shown in Fig. 30-39?