10 Electric Potential

$$V(\vec{b}) - V(\vec{a}) = \int_a^b \vec{E} \cdot \mathrm{d}\vec{l} \qquad V_{point} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad U = qV$$

$$U_{collection} = \frac{\epsilon_0}{2}\int_{\text{Vol}} |\vec{E}|^2 \, \mathrm{d}V = \frac{\epsilon_0}{2}\int_{\text{Vol}} \rho V \, \mathrm{d}V$$

1. A total amount of positive charge Q is spread onto a nonconducting, flat, circular annulus of inner radius a and outer radius b. The charge is distributed so that the charge density per unit area is $\sigma = k/r^3$. Show that (with $V = 0$ at infinity)

$$V = \frac{Q}{8\pi\epsilon_0}\frac{a+b}{ab}$$

at the center of the annulus.

2. The electric field inside a nonconducting sphere of radius R, containing uniform charge density, is radially directed and has magnitude

$$E = \frac{Qr}{4\pi\epsilon_0 R^3}$$

where Q is the total charge in the sphere and r is the distance from the center of the sphere. *(a)* Find the potential V inside the sphere, $r < R$, taking $V = 0$ at $r = 0$. *(b)* What is the difference in electric potential between a point on the surface and the center of the sphere? If Q is positive, which point is at the higher potential? *(c)* Show that the potential at a distance r from the center, where $r > R$, is given by

$$V = \frac{q(3R^2 - r^2)}{8\pi\epsilon_0 R^2}$$

where the zero of potential is taken at $r > R$. Why does this result differ from that of part *(a)*?