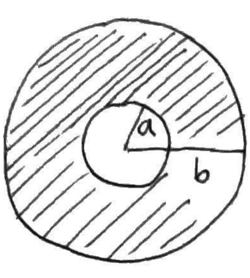
10 Electric Potential

1)



Q

$\sigma = \cancel{K/r^3}$

$$Q = \int \sigma \, dA = 2\pi \int_a^b \frac{K}{r^3} (r\,dr)$$
$$= 2\pi \int_a^b \frac{K}{r^2} dr = 2\pi K \left(\frac{1}{a} - \frac{1}{b}\right)$$

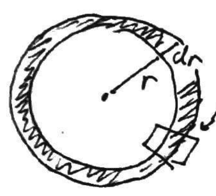
$\sigma = K/r^3$

$$= 2\pi K \left(\frac{b-a}{ab}\right)$$
$$\Rightarrow K = \frac{Q}{2\pi} \cdot \frac{ab}{b-a}$$

To find V, we use <u>superposition.</u>

If $V=0$ at ∞, for each piece of charge dq, $\Rightarrow dV = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r}$

Split annulus to rings of radius r, linear charge density $\lambda = \sigma\, dr$



$dq = \lambda \, dl = r \lambda \, d\theta$

$$\Rightarrow V_{ring} = \int_0^{2\pi} \frac{1}{4\pi\varepsilon_0} \frac{r\lambda\, d\theta}{r} = \frac{\lambda (2\pi)}{4\pi \varepsilon_0} = \frac{\lambda}{2\varepsilon_0}$$

Now integrate the contribution of each ring, using $\lambda = \sigma\, dr = \frac{K}{r^3} dr$

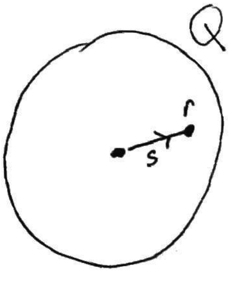
$$V = \int_a^b \frac{\lambda(r)}{2\varepsilon_0} dr = \frac{1}{2\varepsilon_0}\int_a^b \frac{K}{r^3} dr = \frac{1}{2\varepsilon_0}\left(\frac{1}{b^2} - \frac{1}{a^2}\right) \cdot -\frac{K}{2}$$
$$= \frac{K}{4\varepsilon_0}\left(\frac{1}{a^2} - \frac{1}{b^2}\right)$$
$$= \frac{K}{4\varepsilon_0} \frac{b^2 - a^2}{a^2 b^2} = \frac{K}{4\varepsilon_0} \frac{(b-a)(b+a)}{(ab)^2}$$

In terms of Q:

$$V = \frac{Q}{8\pi\varepsilon_0} \frac{ab}{b-a} \cdot \frac{(b-a)(b+a)}{(ab)^2} = \boxed{\frac{Q}{8\pi\varepsilon_0} \frac{b+a}{ab}}$$

✓

2



$$\vec{E} = \frac{Q}{4\pi\varepsilon_0}\frac{r}{R^3}\,\hat{r}$$

(a) $$V(r) - V(0) = -\int_0^r \vec{E}\cdot\hat{r}\,ds \qquad (r<R)$$

$$= -\int_0^r \frac{Q}{4\pi\varepsilon_0}\frac{s}{R^3}\,ds$$

$$= -\frac{Q}{4\pi\varepsilon_0}\frac{1}{R^3}\frac{1}{2}(r^2-0) = -\frac{Q}{4\pi\varepsilon_0}\frac{r^2}{2R^3}$$

Set $V(0)=0 \Rightarrow \boxed{V(r) = -\frac{Q}{4\pi\varepsilon_0}\frac{r^2}{2R^3}}$

(b) $$V(R) - V(0) = -\frac{Q}{4\pi\varepsilon_0}\frac{1}{2R}$$

$Q>0 \Rightarrow$ $V(0)$ at higher potential.

↳ (Also, would have $\vec{E}$ point radially outward ✓)

I misread the problem so can ignore this all.

But result is correct.

~~(c)~~ For $r>R$, $\vec{E} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}\,\hat{r}$

Consider a Gaussian sphere of radius r.

$$\left(\begin{array}{l} Q_{enc} = Q, \quad \Phi = \int \vec{E}\cdot d\vec{A} = \int_S E(r)\,da = E(r)\,4\pi r^2 \\ \Rightarrow E(r) = \Phi/4\pi r^2 \underset{G.L.}{=} \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}\,\hat{r} \end{array} \right)$$

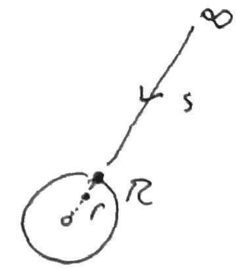
$$\Rightarrow V(r) - V(R) = -\int_R^r ds\,\vec{E}\cdot\hat{r} = -\int_R^r ds\,\frac{Q}{4\pi\varepsilon_0}\frac{1}{s^2}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{r}\right)$$

$$\Rightarrow V(r) = V(R) - \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{r}\right)$$

$$= -\frac{Q}{4\pi\varepsilon_0}\frac{1}{2R} - \frac{Q}{4\pi\varepsilon_0}\frac{1}{R} + \frac{Q}{4\pi\varepsilon_0}\frac{1}{r} = -\frac{3Q}{8\pi\varepsilon_0}\frac{1}{R} + \frac{Q}{4\pi\varepsilon_0}\frac{1}{R}$$

(c) If we instead take $V(\infty) = 0$



$$\Rightarrow V(r) = V(r) - V(\infty) = [V(r) - V(R)] + [V(R) - V(\infty)]$$

$$= -\int_R^r E(s)\,ds - \int_\infty^R E(s)\,ds$$

Outside the sphere, $r > R$,

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r}$$

(Consider Gaussian sphere w/ radius r: $Q_{enc} = Q$, $\Phi = \int \vec{E}\cdot d\vec{A} = E(r)\,4\pi r^2$

$\hookrightarrow E(r) = \Phi/4\pi r^2 = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}$)

$$\Rightarrow V(r) = -\int_\infty^R \frac{Q}{4\pi\varepsilon_0}\frac{1}{s^2}\,ds - \int_R^r \frac{Q}{4\pi\varepsilon_0}\frac{s}{R^3}\,ds$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\left(\frac{1}{R} - \frac{1}{\infty}\right) - \frac{1}{R^3}\frac{1}{2}(r^2 - R^2)\right]$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{r^2}{2R^3} + \frac{1}{2R}\right] = \frac{Q}{4\pi\varepsilon_0}\frac{3R^2 - r^2}{2R^3} = \boxed{\frac{Q}{8\pi\varepsilon_0}\cdot\frac{3R^2 - r^2}{R^3}}$$

Before, we got $V(r) = -\frac{Q}{8\pi\varepsilon_0}\frac{r^2}{R^3}$

These answers are just ~~differents~~ different by a <u>constant offset</u> of the potential.

Before, had $V(0) = 0$. Now, chose $V(\infty) = 0$ and get

$$V(0) = \frac{Q}{8\pi\varepsilon_0}\cdot 3/R$$

So we've just shifted the overall zero pt.

(i.e., before had $V(\infty) = -\frac{Q}{8\pi\varepsilon_0}\frac{3}{R}$)