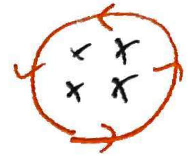
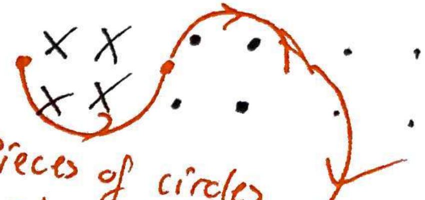
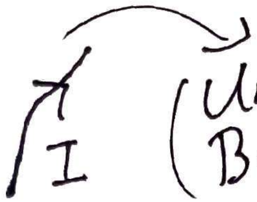
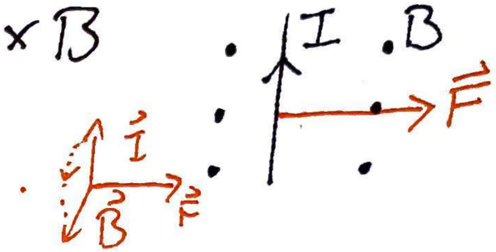
16 Magnetic Force on Wires and Dipoles

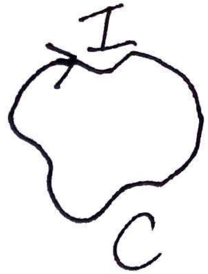
(Thursday: Ampere's Law)

		Force	Motion
• Uniform B-field, $\vec{B}$	moving charged particle $q, \vec{v}$	$\vec{F} = q(\vec{v} \times \vec{B})$	
• Non-uniform B-field	"	"	 "Pieces of circles stuck together"

• Wire/current segment  (Uniform B-field) $\vec{F} = \vec{I} \times \vec{B}$

$d\vec{F} = d\vec{I} \times \vec{B}$



• Closed Loop of current, Uniform B-field 

Net force

$$\vec{F} = \int_C d\vec{F}$$

$$= I \int_C d\vec{\ell} \times \vec{B}$$

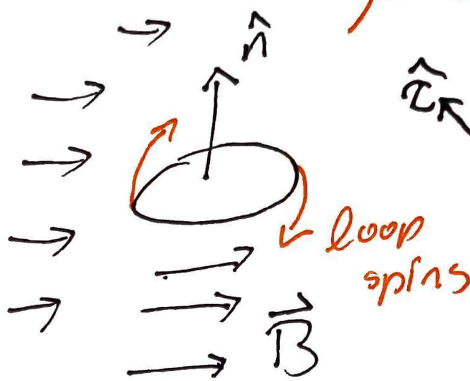
$$= I \underbrace{\left(\int_C d\vec{\ell}\right)}_{0} \times \vec{B} = 0$$

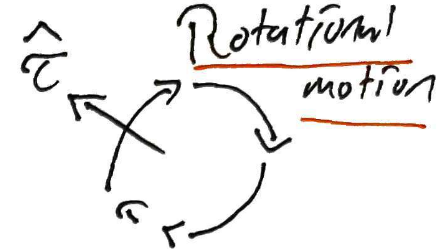
→ No linear motion.

↪ <u>Instead</u> there will be a torque $\vec{\tau}$, makes the loop rotate

$$\boxed{\vec{\tau} = \underbrace{(N_{loops} \cdot A \cdot I)\ \hat{n}}_{\vec{\mu}_{eff}} \times \vec{B}}$$

(Convention for $\hat{n}$.)

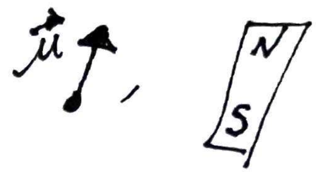




• ☆ Current loops behave like <u>magnetic dipoles</u> (bar magnets)

↪ $\vec{\mu}$

"magnetic (dipole) moment"



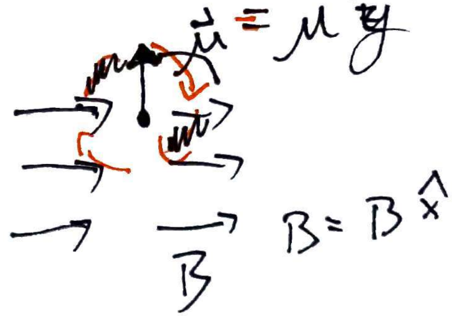
~~Uni~~ Uniform B-field

↪ $\vec{F} = 0$, $\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$

$\vec{\mu} = \mu\hat{y}$

$\vec{\tau} = \mu B(\hat{y} \times \hat{x})$

$= -\mu B(\hat{z})$

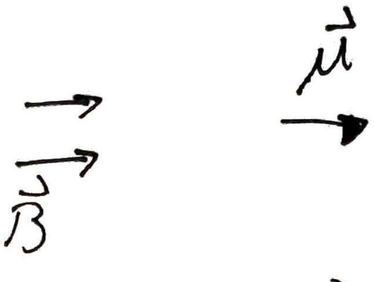


$B = B\hat{x}$

①

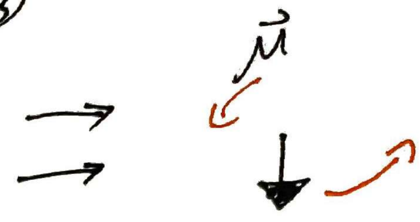
$\tau = -\mu B \hat{z}$

②



$$\vec{\tau} = \vec{\mu} \times \vec{B} = 0$$

③

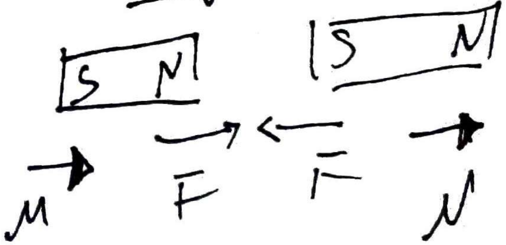


$$\vec{\tau} = \mu B(-\hat{y} \times \hat{x}) = \mu B(+\hat{z})$$

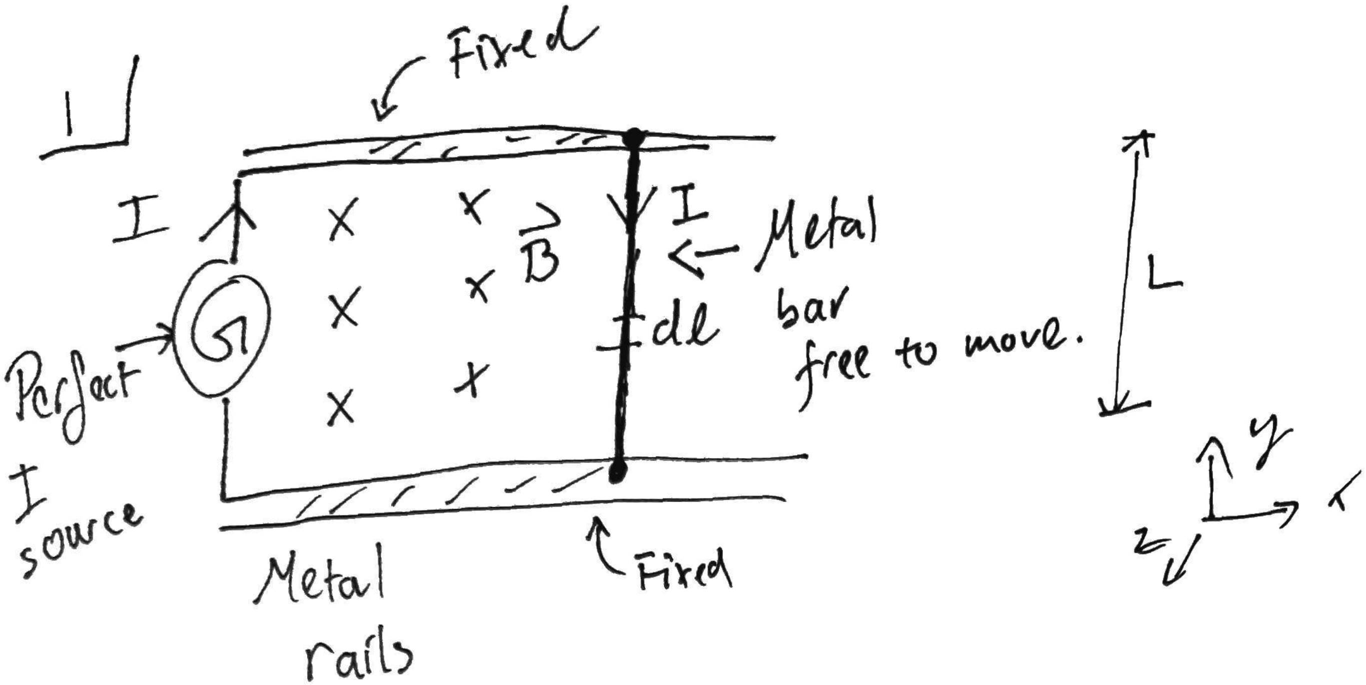
↳ Dipoles will rotate so that they line up with $\vec{B}_{ext}$.

Magnetic dipoles in external B fields act just like Electric dipoles in external E fields.

(*Aside*: Non-uniform B field → $F_z \sim \mu \frac{\partial F}{\partial z}$) * → (Magnetic dipoles *really* do work on eachother)



1



Mass of the bar is M.

$$\vec{a} = \frac{\vec{F}}{M} = \left(\frac{LIB}{M}\right)\hat{x} = \frac{d\vec{v}}{dt}$$

$$v(t) = \left(\frac{LIB}{m}\right) t \,\hat{x}$$

$$KE = \frac{1}{2} m v^2 = \frac{(LIB)^2}{2M} t^2$$

$I \downarrow$

$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$

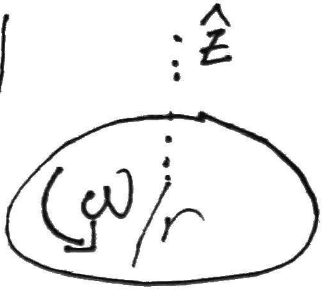
$$= I\,(dl\,(-\hat{y})) \times (-B\hat{z})$$

$$= I B\, dl\; \hat{y} \times \hat{z}$$

$$= I B\, dl\; \hat{x}$$

$$\vec{F} = \int d\vec{F} = \int_0^L dl\; I B\,\hat{x} = LIB\,\hat{x}$$

2]



Charged ring

q uniformly distributed

M total mass

(a) Find I.

(b) Find $\vec{\mu}$

(c) Find $\frac{\mu}{L}$

Angular momentum.

(a) Circumference

$2\pi r$

Charge density

$$\lambda = \frac{q}{2\pi r}$$

$I =$ 

Imagine

Current $I = \dfrac{\text{Amount of charge}}{\text{Time}} =$

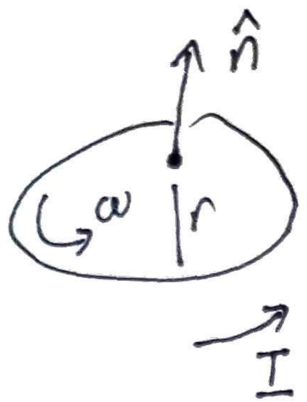
$$\langle I \rangle = q/T$$

$I = \dfrac{q}{T}$ ← period of rotation

$$T = 2\pi/\omega = \frac{2\pi r}{v}$$

$$\boxed{I = \frac{q\omega}{2\pi}} = \frac{qv}{2\pi r} = \lambda v$$

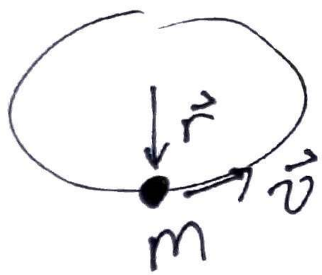
(b) Dipole moment $\quad \vec{\mu} = (I \cdot A \cdot N_{loops})\,\hat{n}$



$$= \frac{q\omega}{2\pi} \cdot \pi r^2 \cdot 1 \cdot \hat{z}$$

$$= \frac{q\omega r^2}{2} = \left(\pi \lambda v r^2 \hat{z}\right)$$

(c) Angular momentum: $\vec{L} = (\vec{r} \times \vec{v})\, m$

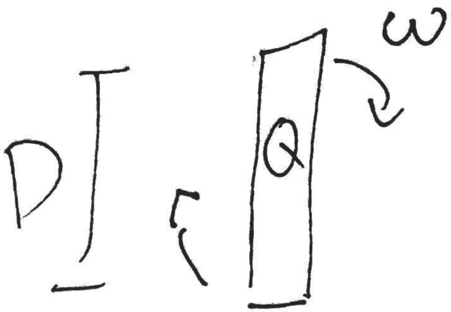


$$v = \cancel{(2\pi r)\omega}\; r\omega$$

This relationship also true for fundamental particles.

$$|L| = r^2 \omega m$$

$$\boxed{\mu/L = \frac{q\omega r^2}{2} \Big/ (\omega r^2 m) = \frac{1}{2}\frac{q}{m}}$$



$$D = 2r$$

Also true!

$$\frac{\mu}{L} = \frac{1}{2}\frac{q}{m}$$

$$\mu = \frac{1}{2}\frac{Q}{m} L$$

$$L = \frac{1}{3}\omega D^2$$
$$= \frac{1}{12}\omega r^2$$

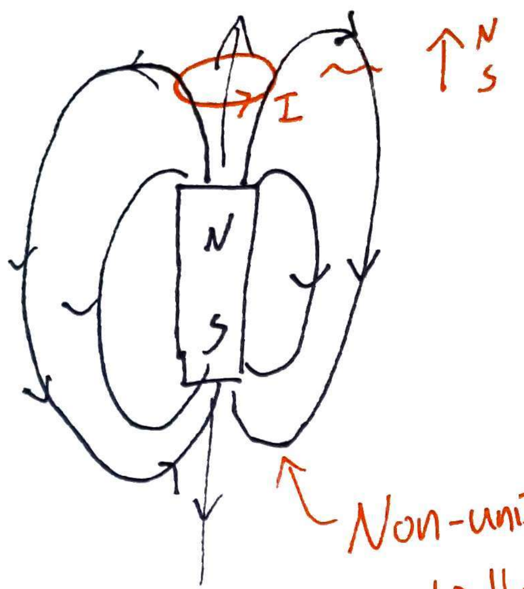
As long as

$$\rho_{mass} \propto \rho_{charge}$$

$$\Rightarrow \boxed{\frac{\mu}{L} = \frac{1}{2}\frac{\rho_{charge}}{\rho_{mass}}}$$

3]

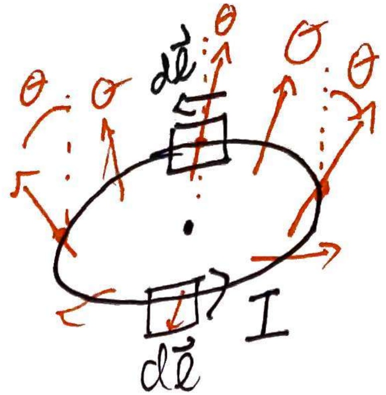
Magnetic field of a dipole / bar magnet



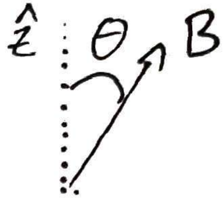
Non-uniform, radially symmetric field

I expect

$\vec{F} = -F\hat{y}$



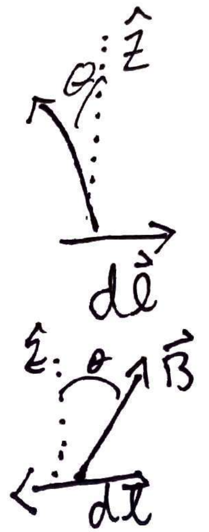
Radius a



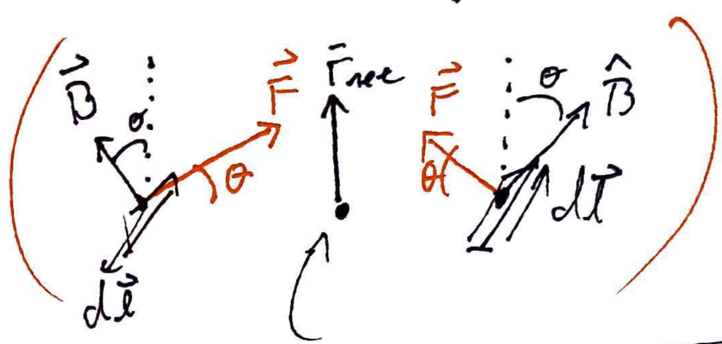
everywhere, radially symmetric way.

(b)

Closed loop, non-uniform field $\Rightarrow$ Can be net F.



$d\vec{F} = \cancel{d}\, I \cancel{B}\, d\vec{l} \times \vec{B}$

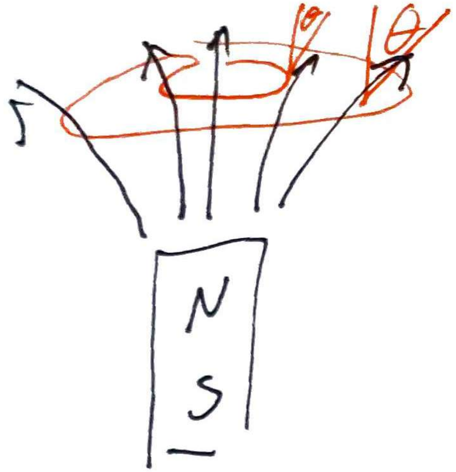


$\boxed{d\vec{F}_{net} = 2 I B \sin\theta \; \hat{z}}$ dl

(opposite sides)

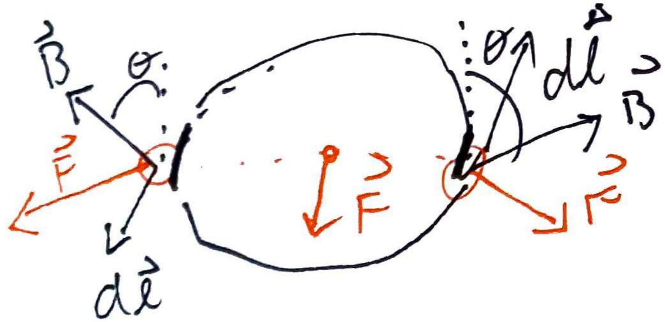
$$\vec{F}_{tot} = \int_{\cap} d\vec{F}_{net} = 2 I B \sin\theta \int_{\cap} dl = \boxed{(2\pi \cancel{R}\, a)\, I B \sin\theta}$$

$$\vec{F}_{tot} = -2\pi a \, I \, B \underbrace{\sin\theta}_{(\sim a)} \hat{z}$$



Expect: $\vec{F}_{tot} = -\cancel{\partial\mu}\; \mu \cdot \frac{\partial B}{\partial z} \hat{z}$

$\pi I B a^2$



$$\Rightarrow \frac{\partial B}{\partial z} = \frac{F_{tot}}{\pi I B a^2} = \frac{2\sin\theta}{a}(-1)$$