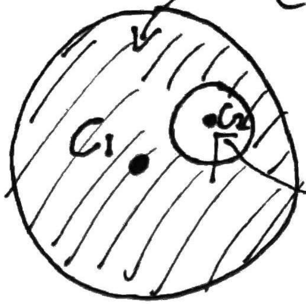
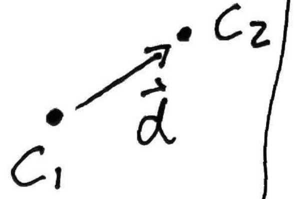
1



Uniformly charged sphere, ρ

Spherical cavity.

Centers C_1, C_2 $\rightarrow$

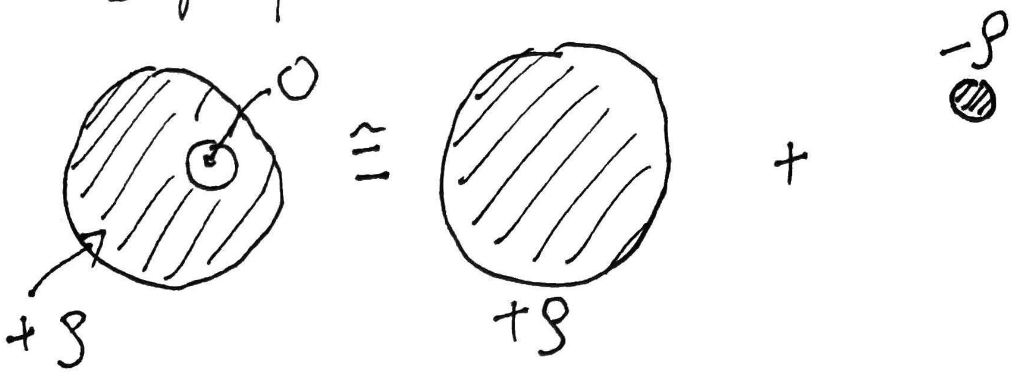


Radii R_1, R_2

Q: Find $\vec{E}$ inside the cavity.

~~(a) First, use Gauss~~

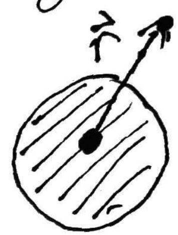
A: Superposition:



$\rightarrow$ At any point in space,

$$\vec{E} = \vec{E}_{\substack{\text{big}\\\text{sphere}\\+\rho}} + \vec{E}_{\substack{\text{small}\\\text{sphere},\ -\rho}}$$

Consider just a single one, say big sphere.

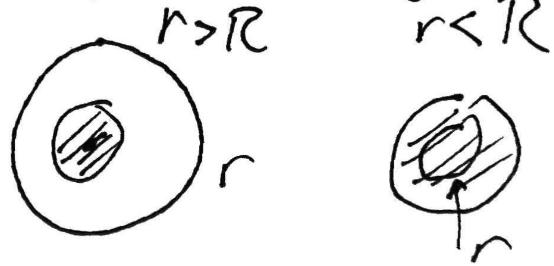


Spherical symmetry

$$\Rightarrow \vec{E}(\vec{r}) = E(r)\,\hat{r}$$

Use Gauss Law:

Spherical surface, radius r:



Flux: $\Phi = \int_S \vec{E} \cdot d\vec{A}$

$= \int_S \underbrace{(\vec{E} \cdot \hat{n})}_{} dA$

$\vec{E} = E(r)\hat{r}$

$\hat{n} = \hat{r}$ for spherical surface

$= \int_S E(r)\, dA$

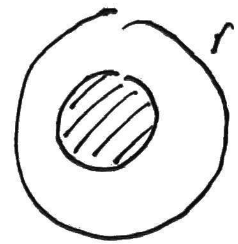
S ← constant radius r

$= E(r) \underbrace{\int_S dA}_{4\pi r^2}$

For *either* $r < R$ or $r \geq R$,

$\Phi(r) = E(r)\, 4\pi r^2$

Charge enclosed:

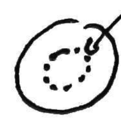


$r \geq R$, all the charge

$Q_{enc} = Q_{tot}$

$= \rho \cdot V_{tot}$

$= \rho \cdot \frac{4}{3}\pi R^3$



$r < R$

$Q_{enc} = \rho \cdot V_{enc}$

$= \rho \cdot \frac{4}{3}\pi r^3$

Gauss' Law:

$\frac{Q_{enc}(r)}{\varepsilon_0} = \Phi(r) = E(r)\, 4\pi r^2$

$\Rightarrow E(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q_{enc}(r)}{r^2}$ ← True for any spherical problem.

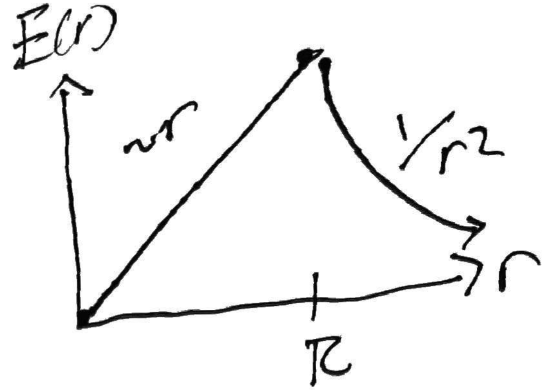
$r \leq R$:

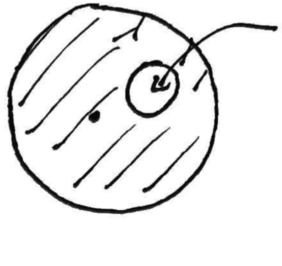
$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\rho\frac{4}{3}\pi r^3$$

$$= \frac{\rho}{3\varepsilon_0} r$$

$$\Rightarrow \boxed{\vec{E}(r) = \frac{\rho}{3\varepsilon_0}\underbrace{r\hat{r}} = \frac{\rho}{3\varepsilon_0}\vec{r}}$$

Special trick for this problem.



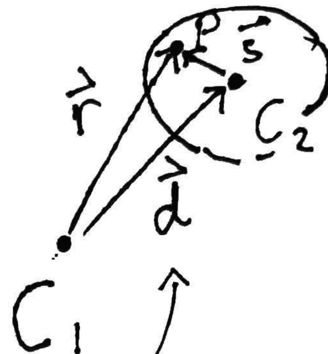


A point in the cavity is inside both of the $+\rho$ sphere, and $-\rho$ sphere.

$r \geq R$:

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\underbrace{\rho\frac{4}{3}\pi R^3}_{Q_{tot}}$$

$$= \frac{Q_{tot}}{4\pi\varepsilon_0}\frac{1}{r^2}$$



$$\vec{E}(p) = \vec{E}_{+\rho}(p) + \vec{E}_{-\rho}(p)$$

$$= \frac{+\rho}{3\varepsilon_0}\vec{r} + \frac{-\rho}{3\varepsilon_0}\vec{s}$$

$$= \frac{\rho}{3\varepsilon_0}(\vec{r} - \vec{s})$$

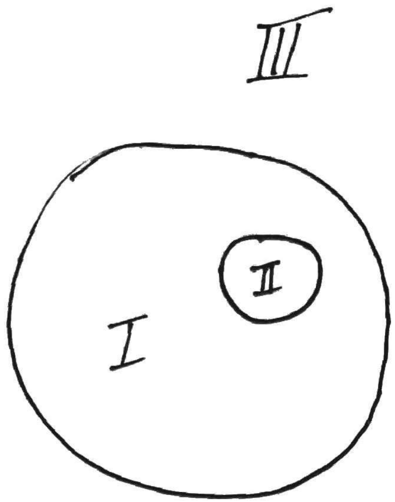
$$= \boxed{\frac{\rho}{3\varepsilon_0}\vec{d}}$$

Uniform field inside cavity!

Vector addition:

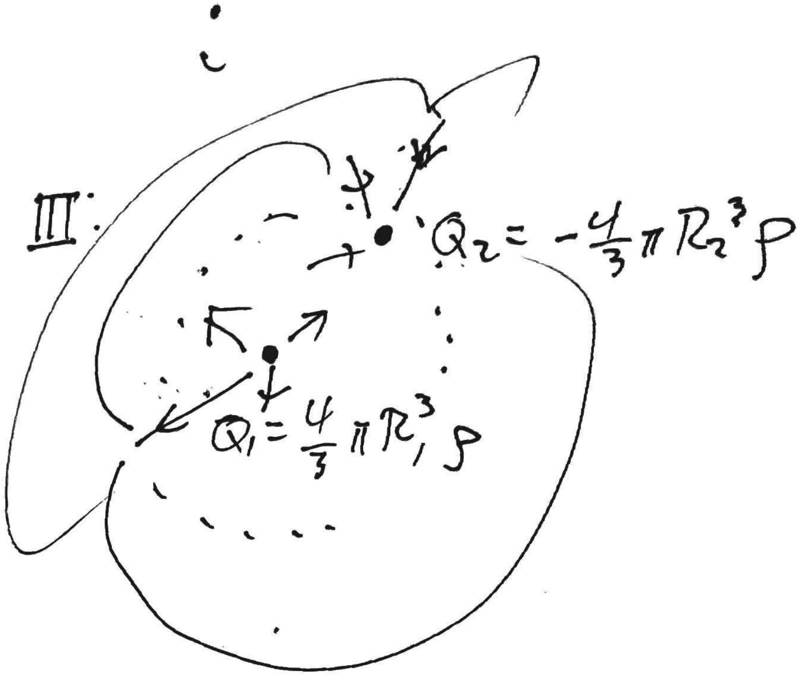
$$\vec{r} = \vec{d} + \vec{s}$$

$$\Rightarrow \vec{d} = \vec{r} - \vec{s}$$

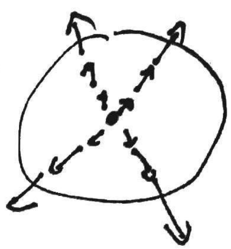


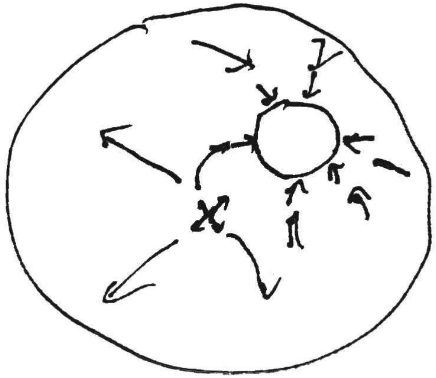
II:

III:

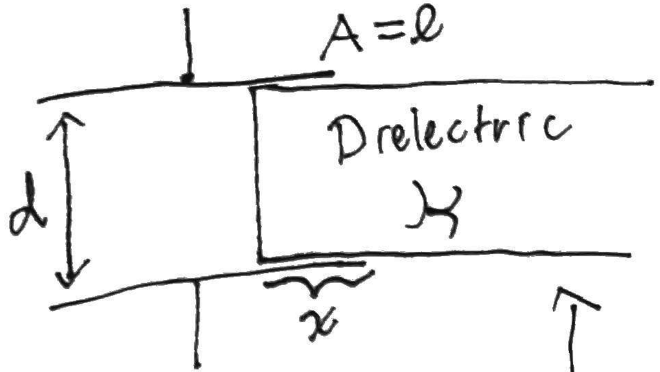


I: $\vec{E}_{-\rho} \sim$ point charge

$\vec{\vec{E}}_{+\rho} \sim$ 

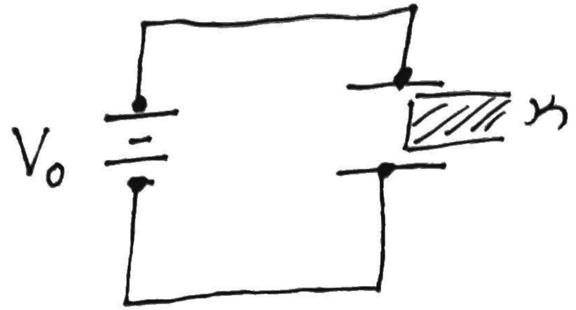


2]



Parallel $\underset{\text{square}}{\uparrow}$ plate capacitor.

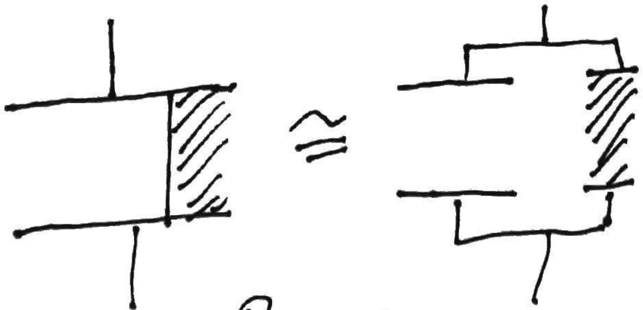
(a) Suppose capacitor remains connected to some battery, V_0



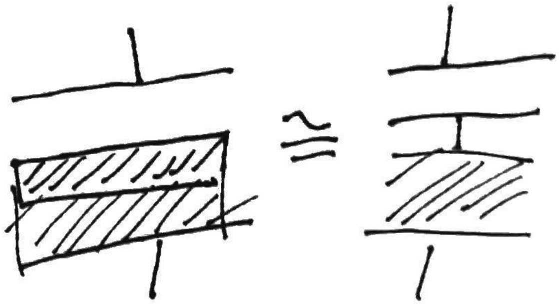
V_0 is fixed.

What is capacitance, $C(x)$?

Partially filled capacitor, is like C in series or parallel:

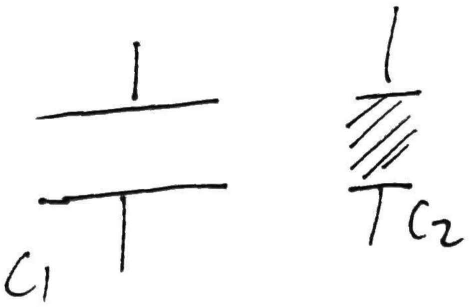


Parallel



Series

How does U change as $f(x)$?



$$U = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$$

$$C = Q/V$$

$Q = CV$

$V = Q/C$

- C is varying
- <u>Either</u> (not both!) Q or V is fixed.

$$C_1 = \varepsilon_0 \frac{A}{d} = \varepsilon_0 \frac{l(l-x)}{d}$$

$$C_2 = (\kappa \varepsilon_0) \frac{A}{d} = \kappa \varepsilon_0 \frac{l x}{d}$$

$$C_{eff} \underset{\substack{\uparrow \\ \text{parall.}}}{=} C_1 + C_2 = \frac{l \varepsilon_0}{d} [l - x + \kappa x]$$

$$\boxed{C_{eff} = \frac{l \varepsilon_0}{d} [l + (\kappa - 1) x]}$$

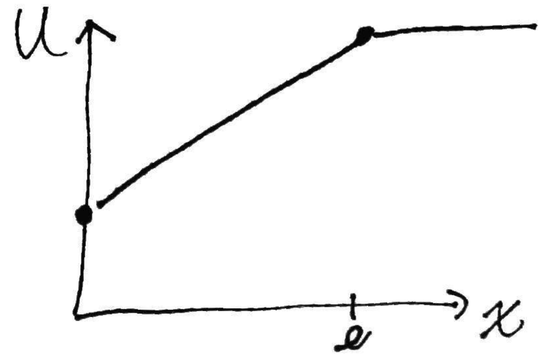
$\uparrow$ $x \leq l$

$$\left(x \geq l \Rightarrow C = \kappa \frac{l^2 \varepsilon_0}{d} \right)$$

(a) V_0 fixed $\leftarrow$ Battery connected

$$\Rightarrow U = \frac{1}{2} C V_0^2$$

$$\boxed{U = \frac{1}{2} \frac{l \varepsilon_0}{d} [l + (\kappa - 1) x] V_0^2}$$



$$\left(x = 0 \rightarrow C = \frac{l \varepsilon_0}{d} [l + 0] = \varepsilon_0 \frac{l^2}{d} = C_0 \right)$$

More dielectric $\Rightarrow$ C larger

V fixed $\Rightarrow$ $U = \frac{1}{2} C V^2$ $\Big] \rightarrow$ More potential energy

$\Rightarrow$ Have to do work to insert the dielectric:

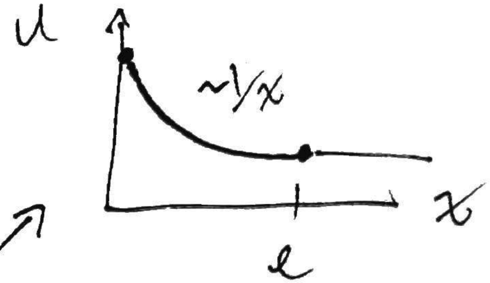
$$W = U(x) - U(0)$$

$$= \frac{1}{2} \frac{\ell \varepsilon_0}{d} (\kappa - 1) x V_0^2$$

<u>Aside:</u> Force on capacitor

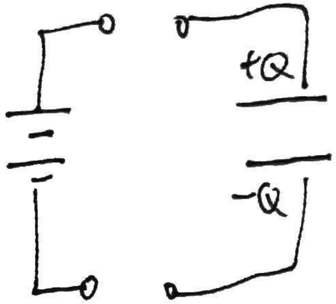
$$\left(F = -\frac{dU}{dx} = \frac{-1}{2} \frac{\ell \varepsilon_0}{d} (\kappa - 1) V_0^2 \right)$$

Repulsive force.



(b) Instead, C initially charged to $\pm Q$.

$\rightarrow$ Disconnect battery <u>before</u> inserting κ.



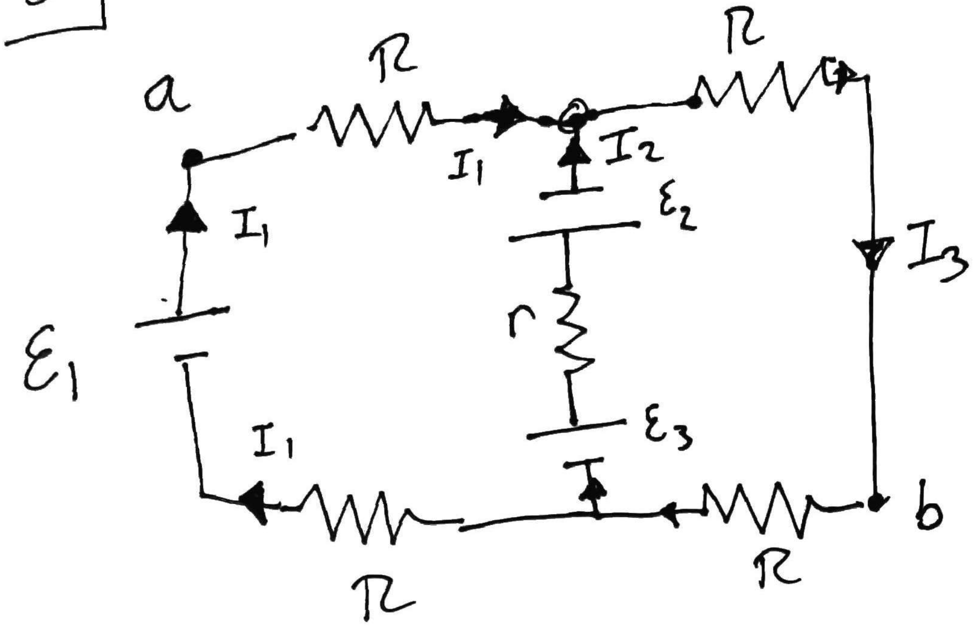
- C is varying
- Q is fixed.

$\Rightarrow U = \frac{1}{2} \frac{Q^2}{C}$ $\Rightarrow$ Less potential energy inside

$\hookrightarrow$ C will <u>do</u> work on dielectric

(Pull it in)

3]



Goal: Find $V(a) - V(b)$

$\varepsilon_1 = 5\ V$

$\varepsilon_2 = 2\ V$

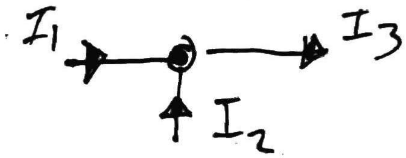
$\varepsilon_3 = 3\ V$

(i) Assign current directions ✓

(ii) Use junction + loop rules to get a bunch of equations

(iii) Solve ↗

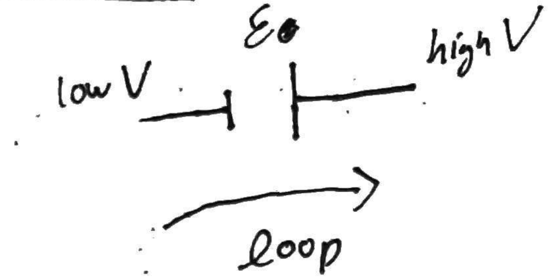
Junction rules:



$$\sum I_{in} = \sum I_{out}$$

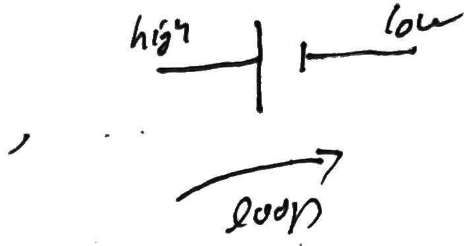
$$\boxed{I_1 + I_2 = I_3}$$

Loop rule:



$+\mathcal{E}$

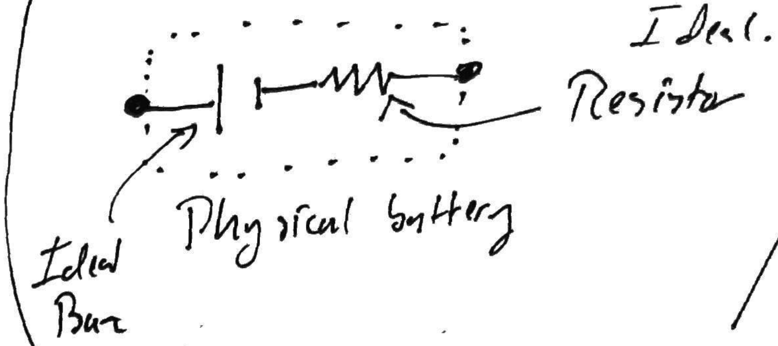
(Gain)

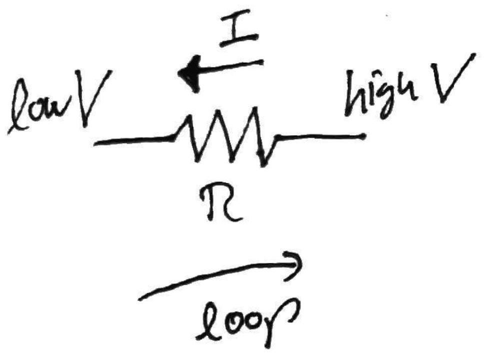


$-\mathcal{E}$

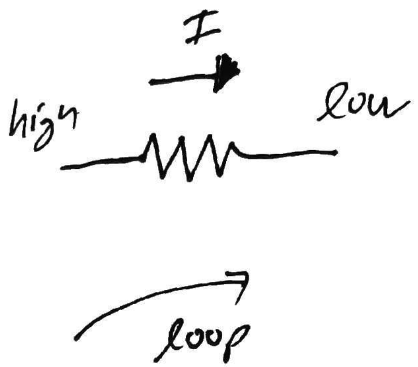
(drop)

Internal resistance:



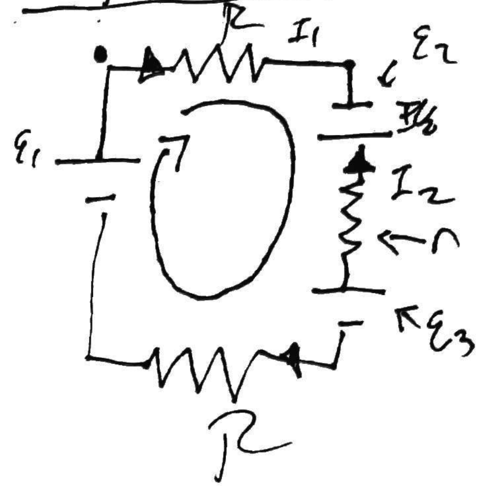


Gain: $+IR$



Drop: $-IR$

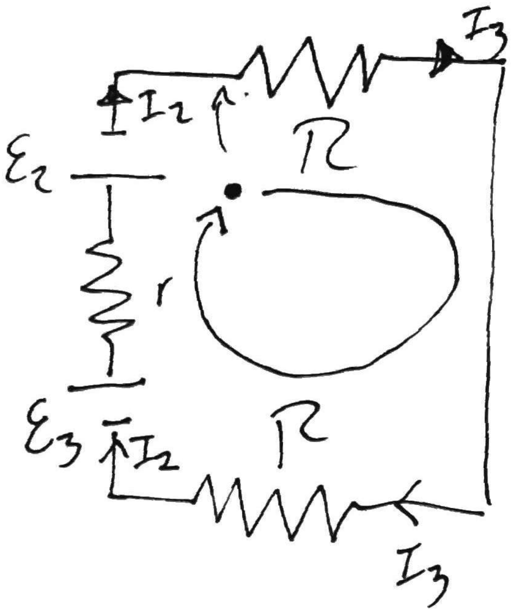
Left loop:



$$0 = -I_1R + \mathcal{E}_2 + I_2 r - \mathcal{E}_3 - I_1R + \mathcal{E}_1$$

$$\boxed{0 = (\mathcal{E}_1 + \mathcal{E}_2 - \mathcal{E}_3) - 2I_1R + I_2 r}$$

Right loop :



$$0 = -I_3 R - I_3 R + \mathcal{E}_3 - I_2 r - \mathcal{E}_2$$

$$\boxed{\mathcal{E}_3 - \mathcal{E}_2 = 2 I_3 R + I_2 r}$$

Left loop: $\mathcal{E}_1 + \mathcal{E}_2 - \mathcal{E}_3 = 2 I_1 R - I_2 r$

Junction: $I_3 = I_1 + I_2$

Use: $\mathcal{E}_1 = 5$, $\mathcal{E}_2 = 2$, $\mathcal{E}_3 = 3$

$$1 = 2 I_3 R + I_2 r = 2 I_1 R + I_2 (r + R)$$

$$4\,V = 2 I_1 R - I_2 r$$

$$1 = 2 I_1 R + I_2 (r+R)$$

$$4 = 2 I_1 R - I_2 r$$

Subtracting:

$$-3 = 0 + I_2 (r+R) + I_2 r$$

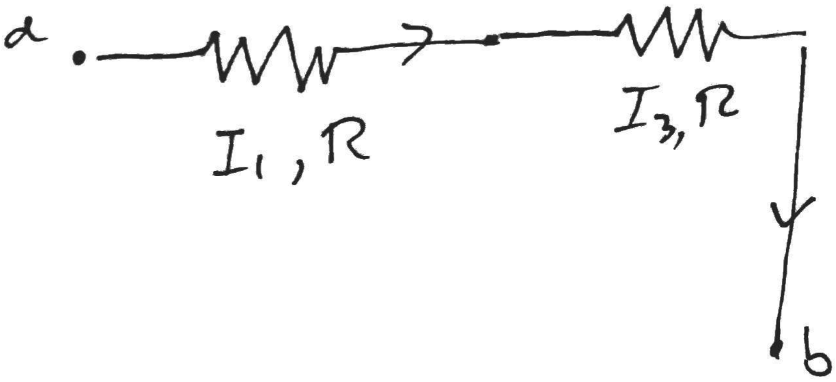
$$\Rightarrow \boxed{I_2 = \frac{-3}{R+2r}}$$

$$\Rightarrow \boxed{\begin{aligned} I_3 &= I_1 + I_2 \\ &= \frac{2}{R} - \left(1 + \frac{r}{2R}\right)\left(\frac{3}{R+2r}\right) \end{aligned}}$$

$$4 = 2 I_1 R - \left(\frac{-3}{R+2r}\right) r$$

$$\boxed{I_1 = \frac{1}{2R}\left[4 - \frac{3r}{R+2r}\right]}$$

$$= \frac{2}{R} - \frac{3r}{2R(R+2r)}$$



$$I_1 + I_1 + I_2 = 2I_1 + I_2$$

$$V(a) - V(b) = I_1 R + I_3 R = (\overbrace{I_1 + I_3}) R$$

$$\boxed{= \frac{1}{2}\left[4V - 3V\frac{r}{R+2r}\right] + 2V - \left(1 + \frac{r}{2R}\right)\left(\frac{3R}{4R+2r}\right)V}$$