15 Magnetic Force on Charges / Dipoles

Recall:

$\oplus \longleftrightarrow \ominus \leftarrow q_2$

q_1 $F_{coulomb}$ – stationary charged particles

$q_1 \xrightarrow{\text{creates}} \vec{E} \xrightarrow{\text{exerts a force on}} q_2$

$\left(\vec{E}_1 = \frac{kq}{r^2}\right)$ $\left(\vec{F}_2 = q_2 \vec{E}_1\right)$

Additionally: Moving charges experience an additional force

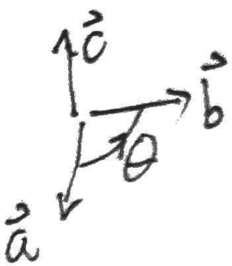
$\oplus \downarrow v$ $\quad$ $\uparrow v$ $\oplus$

$\vec{F}_{net} = \vec{F}_{coulomb} + \vec{F}_{mag}$ ← depends on v

Moving $q_1 \xrightarrow{\text{create}} \vec{B} \xrightarrow{\text{exert a force on}} q_2$

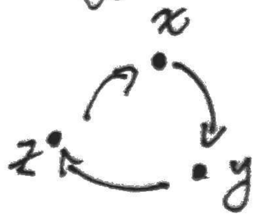
$$\boxed{\vec{F} = q_2(\vec{v} \times \vec{B})}$$

Review - Cross product rules

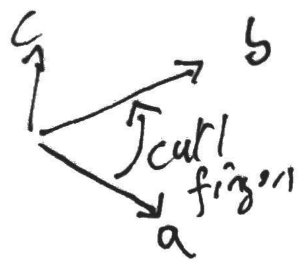


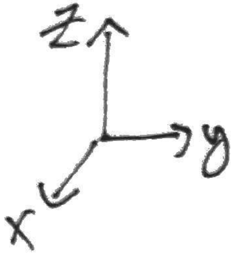
$\vec{c} = \vec{a}\times\vec{b}$, $\vec{c}\perp\vec{a}$, $\vec{c}\perp\vec{b}$, $|c| = a\,b\sin\theta$ (General)

Unit vectors: $\hat{x}\times\hat{y} = \hat{z}$, $\hat{y}\times\hat{z} = \hat{x}$, $\hat{z}\times\hat{x} = \hat{y}$



"Right hand rule"





Opposite order:

$$\boxed{\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}}$$

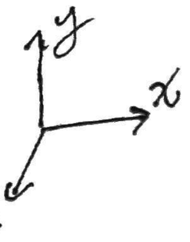
Caution!!

↳ $\hat{y}\times\hat{x} = -\hat{z}$, $\hat{z}\times\hat{y} = -\hat{x}$, $\cancel{y}$ $\boxed{\hat{x}\times\hat{z} = -\hat{y}}$

Example: Motion of a charged particle in a uniform B-field.

q, $\vec{v}(t=0) = v_0 \hat{x}$

$\vec{B} = B\hat{z}$

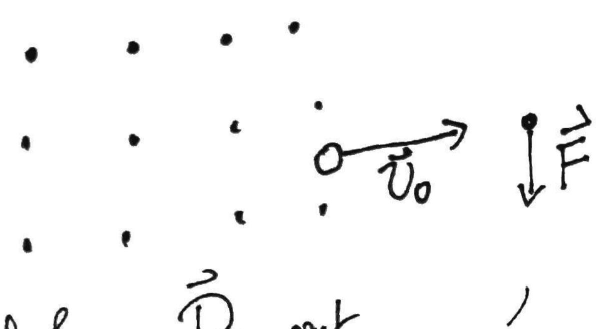


(out of the page)

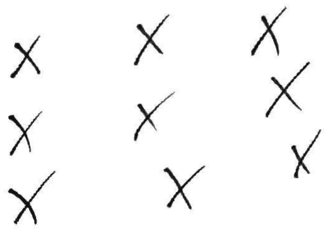
($\vec{B} \sim$ 

Head on: $\bullet$ pointed

Behind: $\times$ feather tail)



Uniform $\vec{B}$ out of page $(+\hat{z})$

( $\vec{B}$ into page)

Initially, $\vec{v}(0) = v_0 \hat{x}$

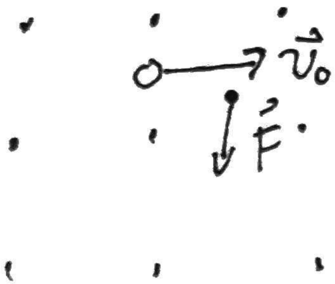
$$\Rightarrow \vec{F} = q\,\vec{v} \times \vec{B}$$

$$= q(v_0 \hat{x}) \times (B\hat{z})$$

$$= q v_0 B \underbrace{(\hat{x} \times \hat{z})}_{-\hat{y}}$$

$$= q v_0 B(-\hat{y})$$

Initially:



What about later times?

- Could find $\vec{v}(t)$, $\rightarrow \vec{F}(t) \rightarrow \vec{a}(t)$

(Newton's 2^{nd} Law)

- Notice:

For any charged particle moving in $\vec{B}(\vec{r})$

Because $\vec{F} \perp \vec{v}$ $\quad (\vec{F} \propto \vec{v} \times (\vec{B}))$

↳ Magnetic force will do <u>no work</u>. $W = \vec{F} \cdot \Delta\vec{x} = 0$

→ Kinetic energy is const.

$$KE = \frac{1}{2} m v^2 \Rightarrow \boxed{|\vec{v}| = \text{const.}}$$

Recall from 7A → ☆ Leads to uniform circular motion.

↑ Use that $|\vec{F}| = q\,|\vec{v} \times \vec{B}| = q\, v B \underbrace{\sin\theta}_{1}$

Magnetic: $|F| = qvB$

Circular motion: $|F| = m a_c = \frac{mv^2}{r}$

$\hat{y}\times\hat{z} = \hat{x}$

$F = q\vec{v}\times\vec{B} \sim -\hat{y}\times\hat{z} = -\hat{x}$

$F \sim (-\hat{x})\times\hat{z} = \hat{y}$

$\Rightarrow \quad qvB = \frac{mv^2}{r}$

$\Rightarrow$

$mv \sim$ initial momentum $\rightarrow$ harder to pull charged particle $\rightarrow$ larger circle

$* \quad \boxed{r = \frac{mv}{qB}}$

Stronger $\vec{B}$, or larger q $\Rightarrow$ More force $\Rightarrow$ Tighter circle

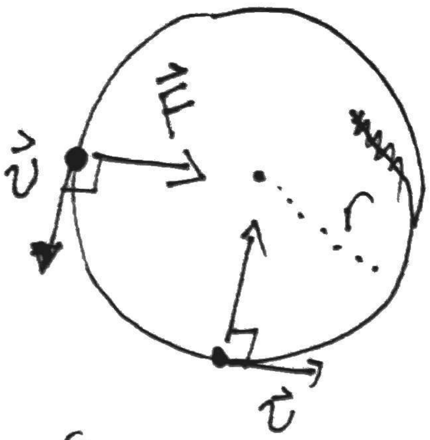
Angular frequency

Doesn't depend on v

$* \quad \boxed{\omega \equiv \frac{v}{r} = \frac{qB}{m} \quad \text{"Cyclotron frequency"}}$

(Period: $T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$)

Uniform circular motion:



$\vec{F}$ perpendicular to $\vec{v}$.

$|F|$ constant.

r?

Newton's second law

$$|F| = m a$$

"Centripetal acceleration" towards the center

Mnemonic / Argument

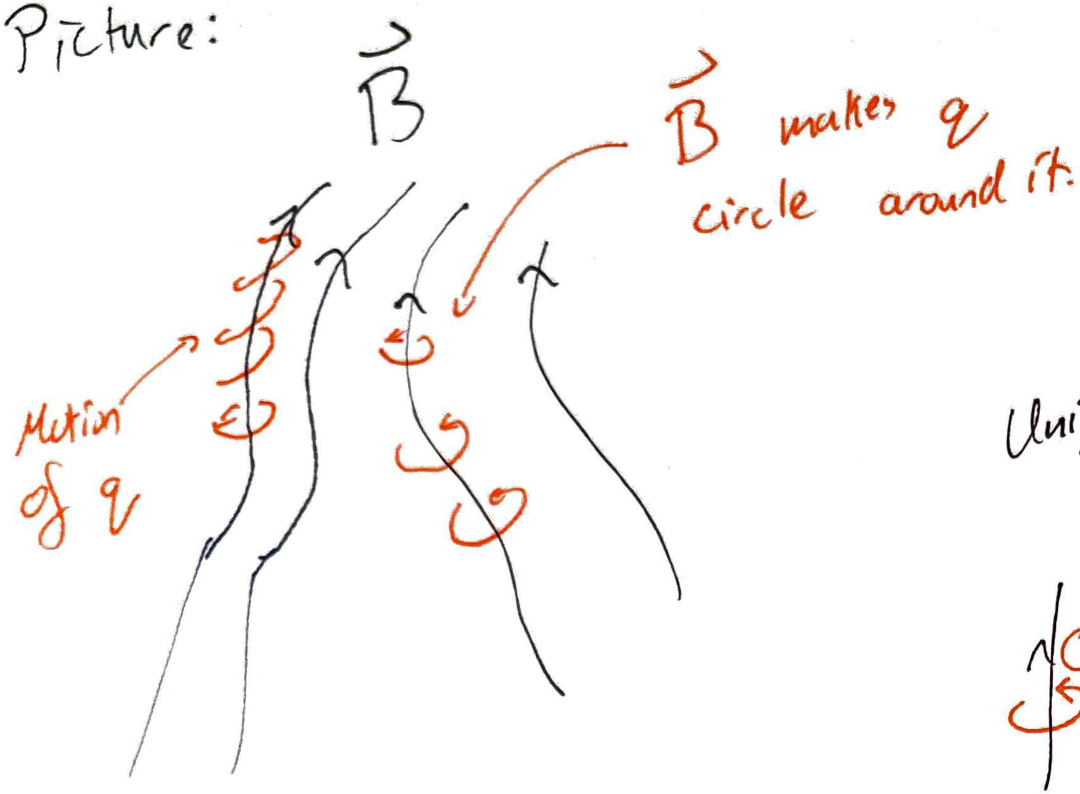
Dimensional Analysis:

$$[a] = \frac{[\text{distance}]}{[\text{time}]^2} = \left(\frac{[\text{distance}]}{[\text{time}]}\right)^2 \cdot \frac{1}{[\text{distance}]}$$

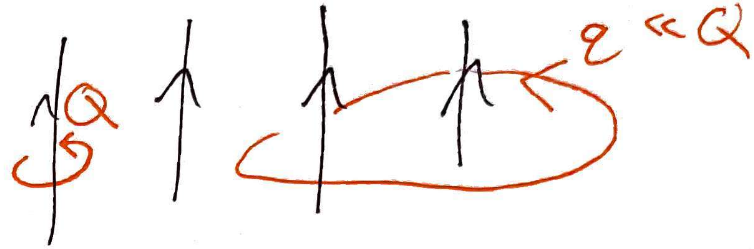
$$= ([v])^2 \cdot \frac{1}{[r]}$$

$$\Rightarrow \boxed{a_c = \frac{v^2}{r}}$$

Picture:

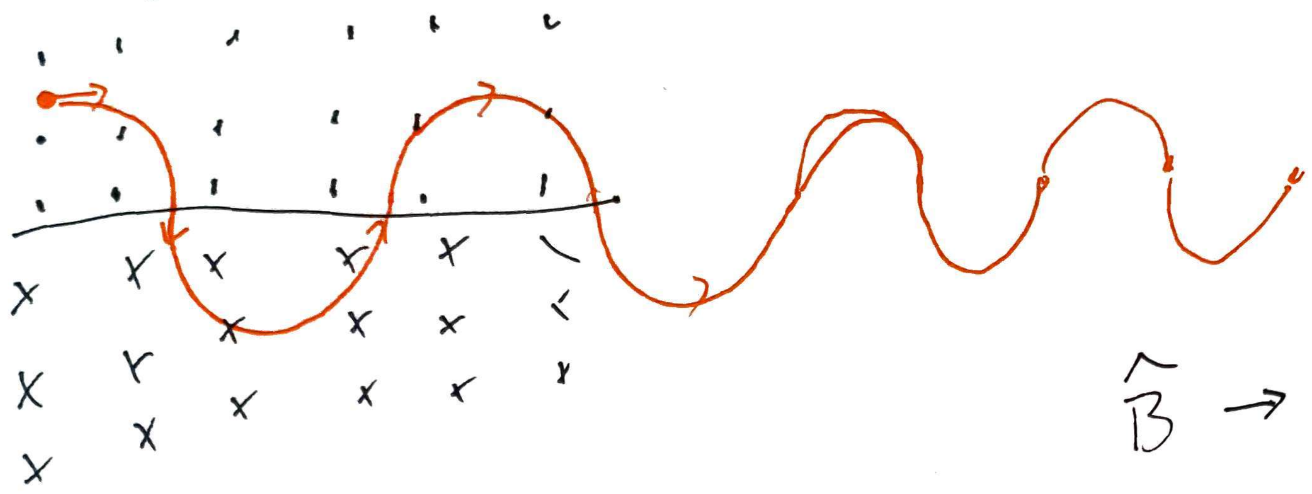


Uniform:



Non-uniform:

Changing $\vec{B}$ direction:



$\hat{B} \rightarrow$ CW or CCW

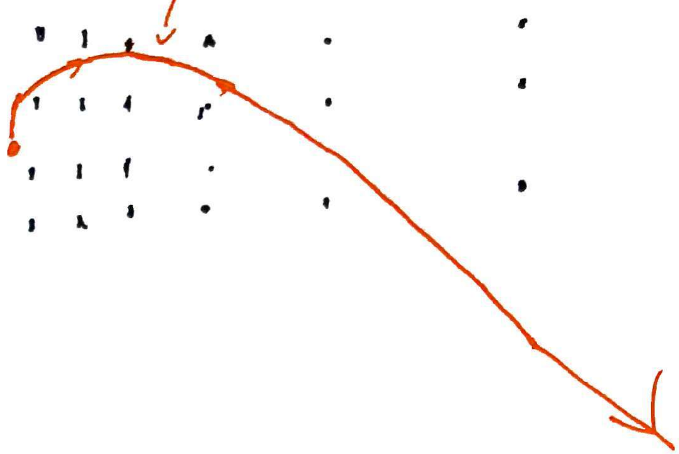
$|B| \rightarrow$ Larger or smaller circles

(curvature)

Changing $|B|$?

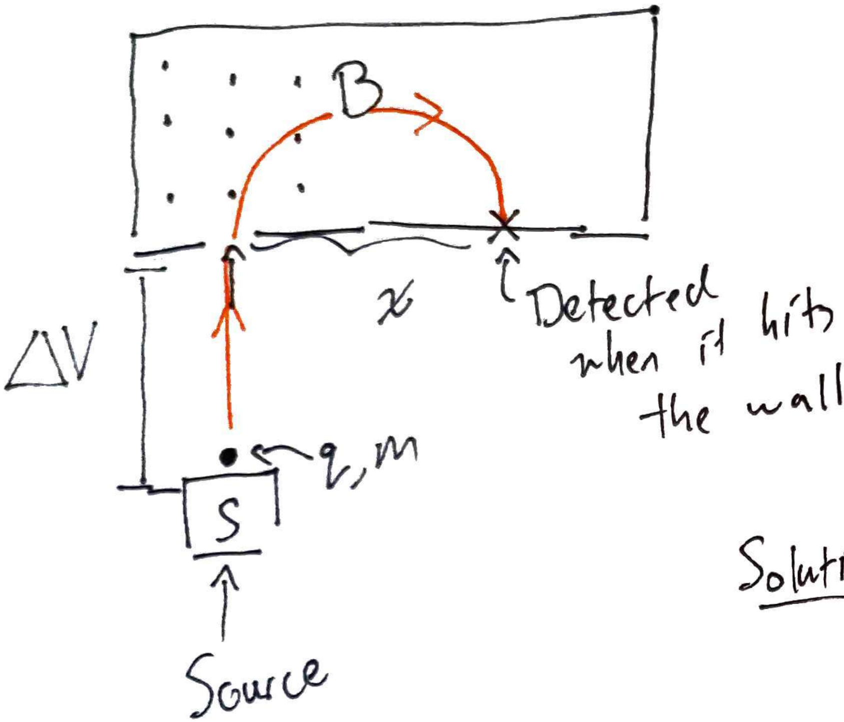
$\sim 2B$

F is lower $\rightarrow$ r ~~small~~ larger



$B=0$

Application / Problem: Mass spectrometer.



If I know:

- q
- B
- ΔV
- x

$\Rightarrow$ Measure m?

Solution: - Uniform B field
$\rightarrow$ Uniform circular motion

$$qvB = F = m a_c = \frac{mv^2}{r} \quad r = x/2$$

$$\Rightarrow \left(m = \frac{qvB}{v^2} \cdot r = \frac{qB}{v} r = \frac{qBx}{2v} \right)$$

$$KE = q\,\Delta V = \frac{1}{2} m v^2 \Rightarrow v^2 = \frac{q\,\Delta V}{m}\cdot 2$$

accelerates up ΔV

$$\left.q v B = \frac{m v^2}{r}\right\} \Rightarrow m = \frac{qBr}{v}$$

$$\Rightarrow m^2 = \frac{q^2 B^2 r^2}{v^2}$$

$$m^2 = \frac{q^2 B^2 r^2}{\left(q \frac{\Delta V}{m} 2\right)}$$

"Macroscopic info"

$$r = x/2$$

$$m^2 = \frac{B^2 q}{\Delta V}\,\frac{r^2}{2}\cdot m$$

"Atomic information"

$$\boxed{\frac{q}{m} = \frac{8\,\Delta V}{B^2 x^2}}$$

$$\boxed{m = \frac{B^2 q\, x^2}{8\,\Delta V}}$$