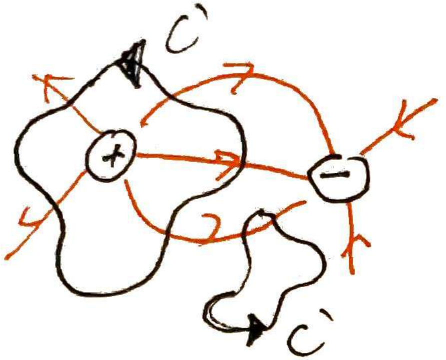
Faraday's Law, Motional EMF

• In electrostatics



$$\oint_C \vec{E}\cdot d\vec{l} = 0$$

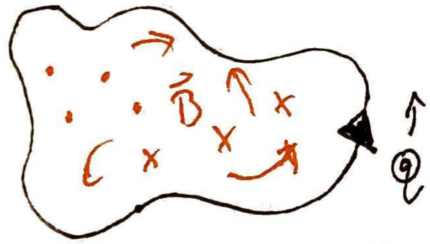
Electric field has no "curl" in a static configuration.

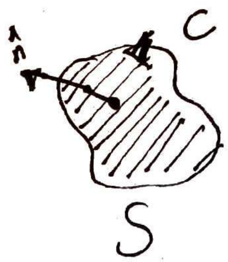
• Faraday's Law

$$\overbrace{\oint_C \vec{E}\cdot d\vec{l}}^{\mathcal{E}_{ind}} = -\frac{d\Phi_B}{dt}$$

"Lenz Law"

"Induced EMF"





$$W = -\oint_C \vec{F}\cdot d\vec{l}$$
$$= -q \oint_C \vec{E}\cdot d\vec{l}$$
$$= -q\, \mathcal{E}_{ind}$$

Define magnetic flux

$$\Phi_B \equiv \int_S \vec{B}\cdot d\vec{A}$$

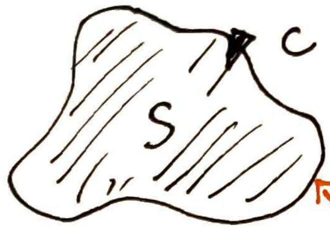
• How to get $\frac{d\Phi_B}{dt}$?

② Changing angle b/w $\hat{B}, \hat{n}$

$\vec{B} \cdot \hat{n} = B \cancel{\sin\theta} \cos\theta$

$$\Phi_B = \int_S \overbrace{(\vec{B} \cdot \hat{n})} \, dA$$

① $B(t)$

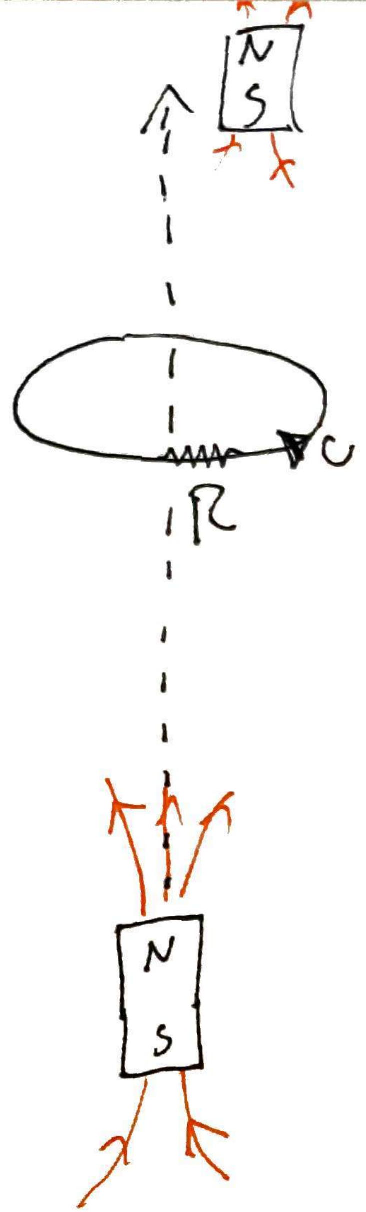


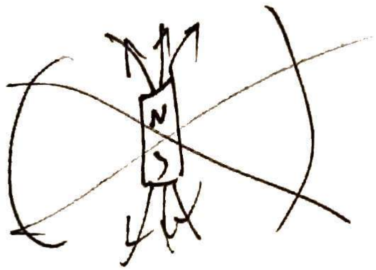
③ Change the area enclosed.

(Make C bigger or smaller.)

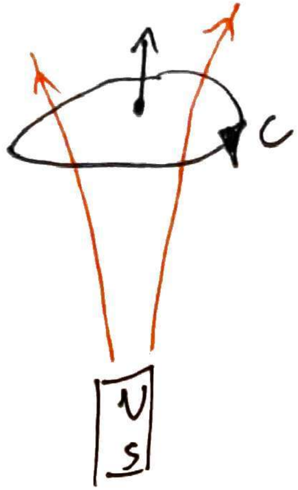
Regard C as a ~~piece~~ loop of wire, which can move.

1]





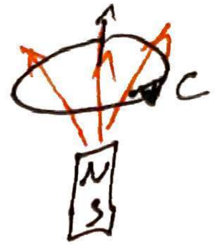
(i)



$\Phi_B > 0$

Φ_B small

(ii)



$\Phi_B > 0$

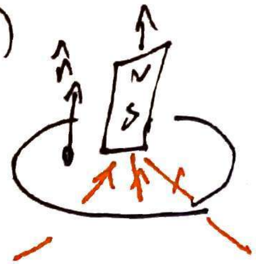
Φ_B large

(iii)

$\Phi_B \simeq 0$

(maybe ≤ 0)

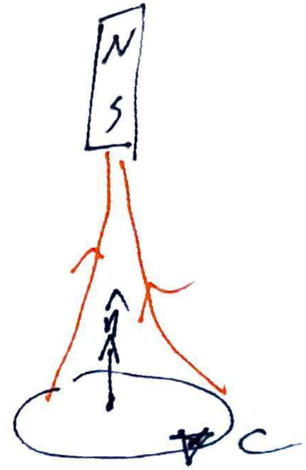
(iv)



$\Phi_B > 0$

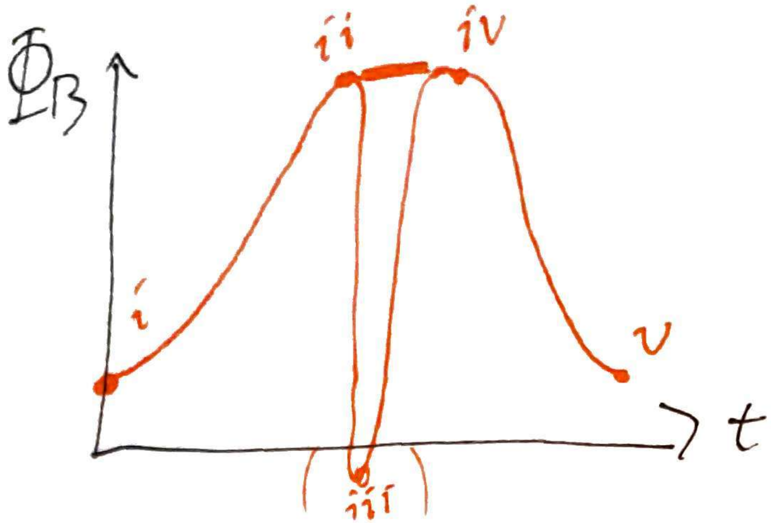
Φ_B large

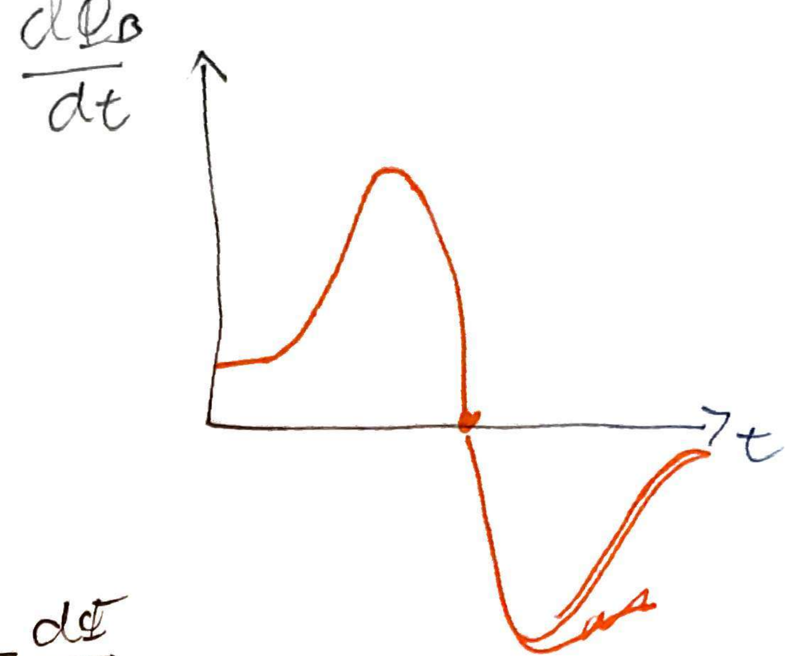
(v)

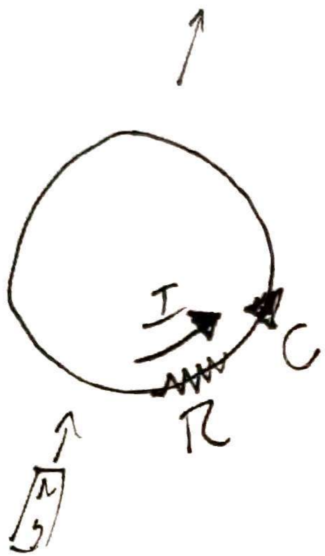


$\Phi_B > 0$

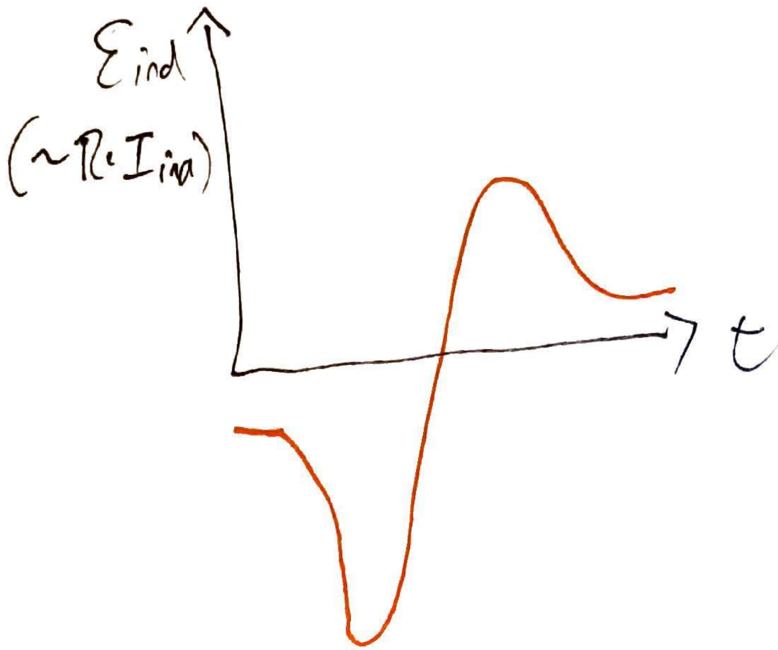
Φ_B small





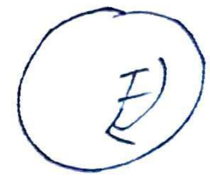


$$\mathcal{E}_{ind} = \oint_C \vec{E}\cdot d\ell = -\frac{d\Phi}{dt}$$



Initially:

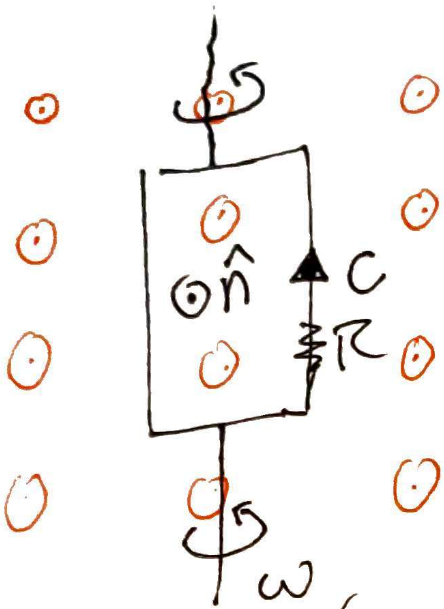
$\frac{d\Phi_B}{dt} > 0$



Later

$\frac{d\Phi_B}{dt} < 0$

2) Constant uniform $\vec{B}$, out of ~~B~~ page



Rotating wire frame (A const)

↳ $\vec{B}\cdot\hat{n}$ is changing.

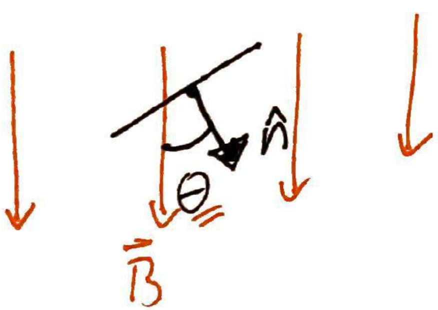
$$\Phi_B = \int_S (\vec{B}\cdot\hat{n})\, dA$$

$$= \cancel{\int_S} (\vec{B}\cdot\hat{n}) \int_S dA$$

$$= B \underline{\underline{\cos\theta}} \cdot A$$

↳ $\left(\theta = \omega t \mod 2\pi\right)$

Top-down:



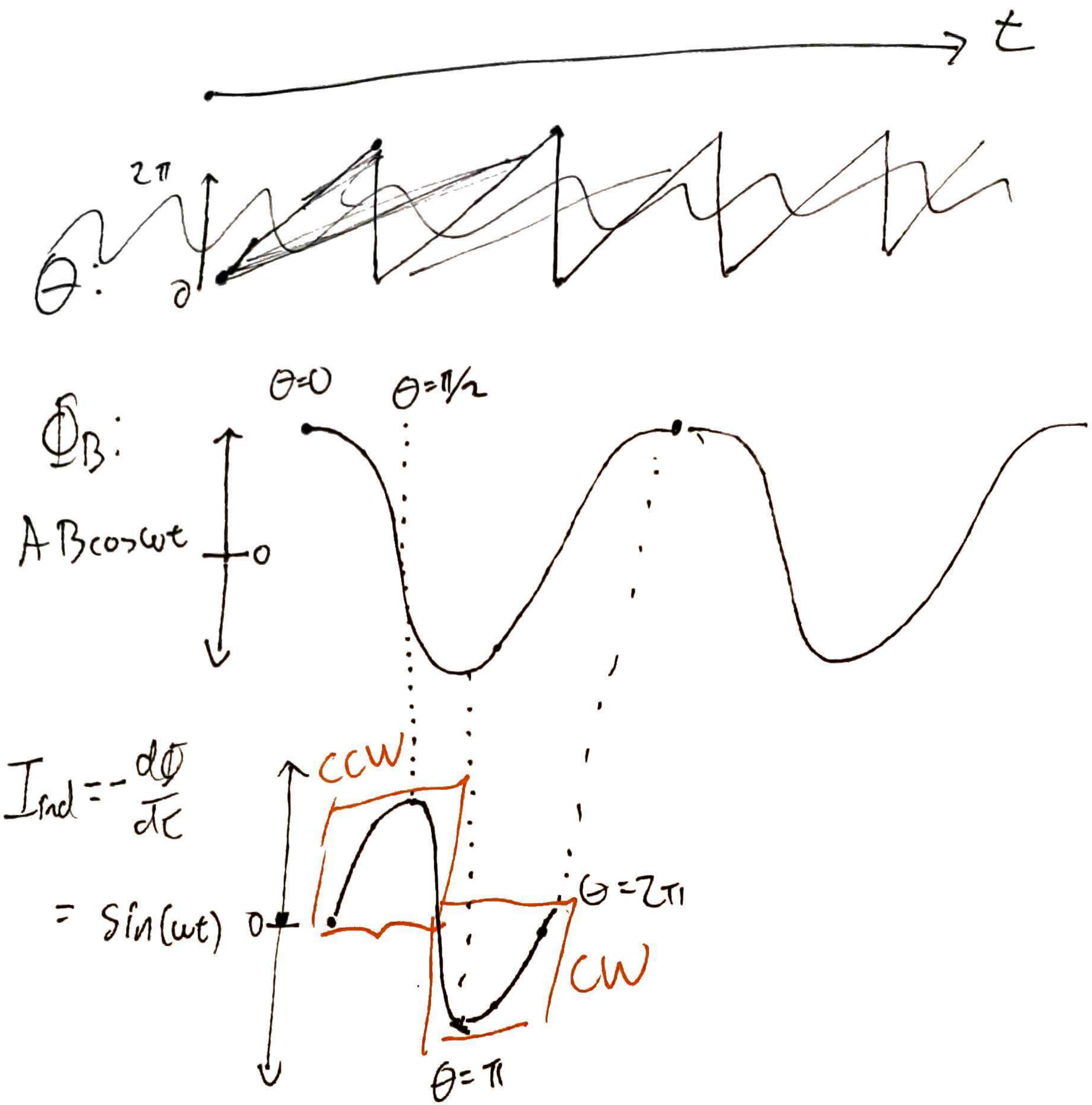
$$\frac{d\Phi_B}{dt} = BA \frac{d}{dt}\cos\theta = -BA\sin\theta \frac{d\theta}{dt}$$

$$= -BA\omega \sin\theta(t)$$

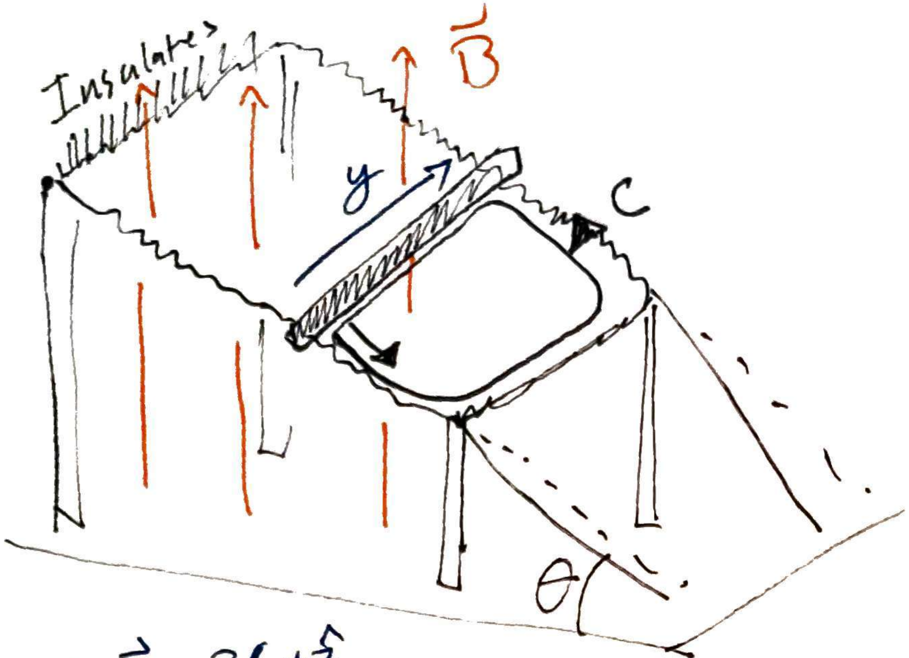
$$\mathcal{E}_{ind} = -\frac{d\Phi_B}{dt} = +BA\omega \sin\theta(t)$$

$$= +BA\omega \sin(\omega t)$$

$$R \Rightarrow I_{ind} = \mathcal{E}_{ind}/R = -\frac{1}{R}\frac{d\Phi_B}{dt}$$


t
2π
θ:
0
θ=0
θ=π/2
Φ_B:
A B cos ωt
0
I_ind = - dΦ/dt
= sin(ωt)
0
CCW
CW
θ=2π
θ=π

3]



Gravity will pull bar down ramp.

↳ Area of C decreases

↳ $\frac{d\Phi_B}{dt} < 0 \Rightarrow \mathcal{E}_{ind}$, & I_{ind}

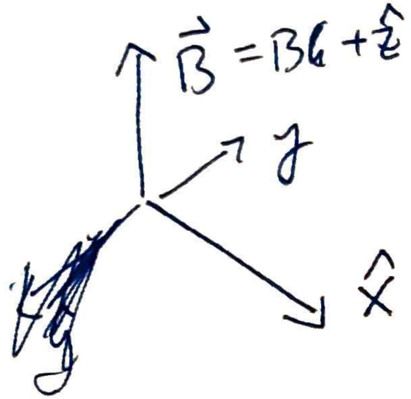
Bar: m, R, L

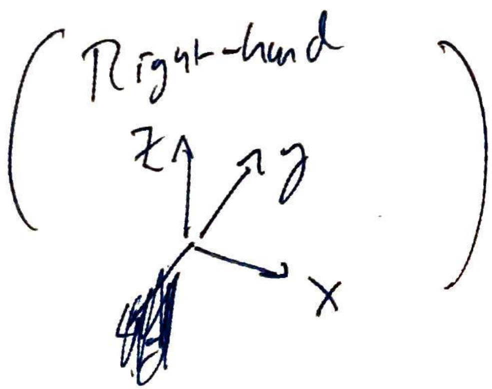
★ $\vec{B}$ will exert a Lorentz force on this.

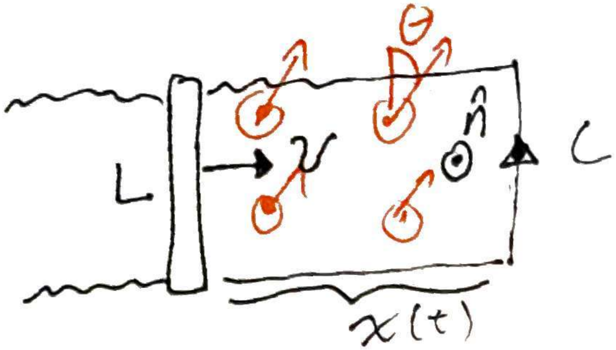
F_L

Resist motion of bar ("magnetic friction")

(Terminal velocity)







$$A_C(t) = L \cdot x(t)$$

$$\Phi_B = \int_A \vec{B} \cdot d\vec{A} = \int (\vec{B} \cdot \hat{n})\, dA$$

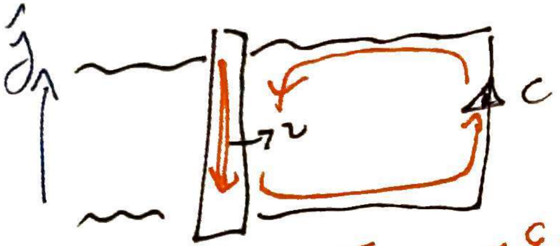
$$= B\cos\theta \int dA$$

$$= B\cos\theta \; A(t)$$

$$= B\cos\theta \; L \, x(t)$$

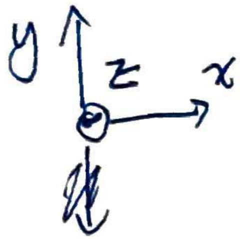
$$\frac{d\Phi_B}{dt} = B\cos\theta L \frac{d}{dt} x(t)$$

$$= - v B L \cos\theta$$



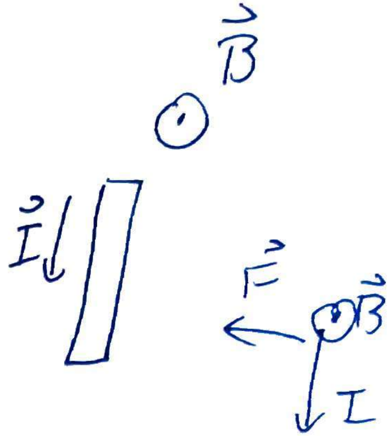
$$\mathcal{E}_{ind} = -\frac{d\Phi_B}{dt} = + \underset{\substack{\uparrow \\ (v(t))}}{v} L B \cos\theta$$

$$I_{ind} = \mathcal{E}_{ind}/R$$



$$|I_{ind}| = \frac{|\mathcal{E}_{ind}|}{R} = \frac{vLB}{R}\cos\theta$$

$$\vec{I}_{ind} = \left(\frac{vLB}{R}\cos\theta\right)\cdot(-\hat{y})$$

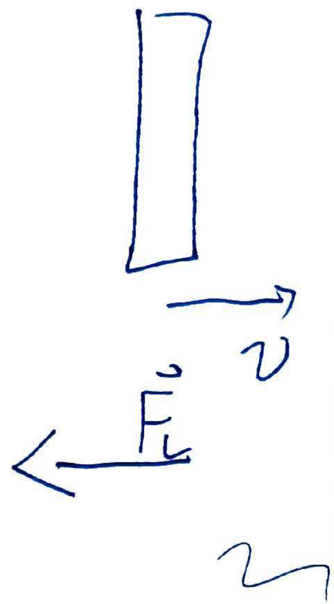


Lorentz force b/w $\underline{\underline{\vec{B}_{ext}}}$, $\underline{\underline{\vec{I}_{ind}}}$

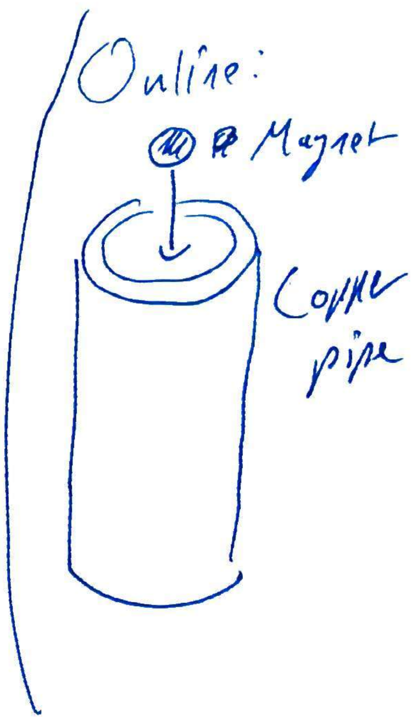
$$\vec{F} = (L\,\vec{I}_{ind}) \times \vec{B}_{ext}$$

$$= \frac{vL^2}{R} B\,(-\hat{y}) \times (B\hat{z})$$

$$= \frac{vL^2B^2}{R}(-\hat{x})$$

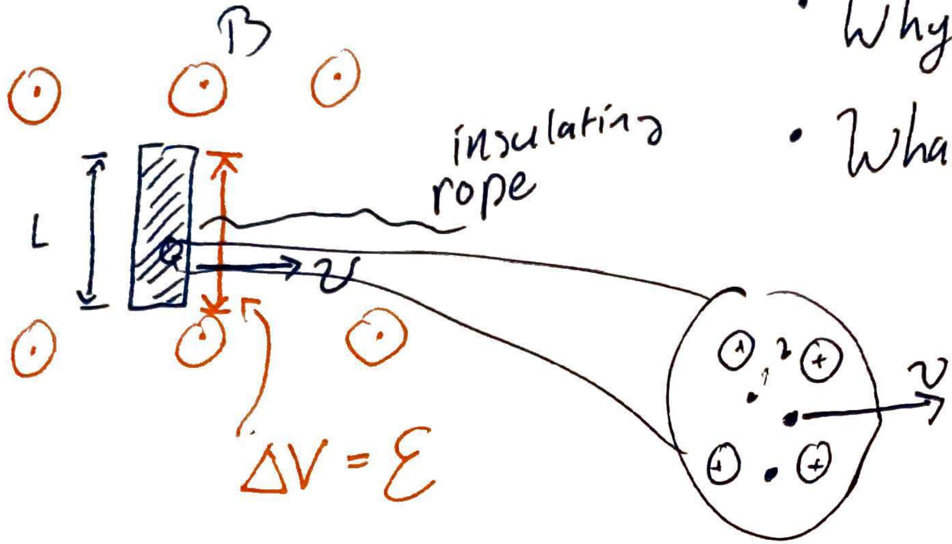


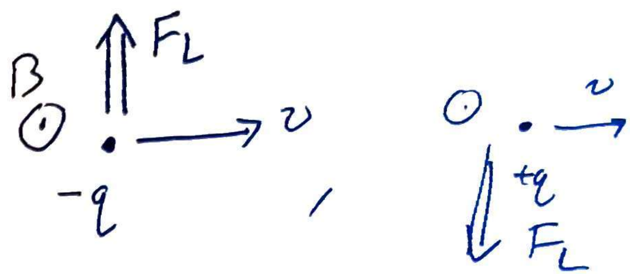
$\boxed{\star\ \vec{F} \text{ is opposite to } v}$



Solid conductor moving in a const. $\vec{B}$-field

- Why there $\mathcal{E}$?
- What is its magnitude?

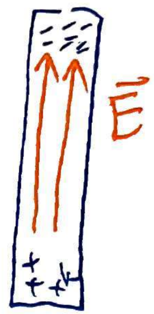




Conductor:

✱ A material, where the charges (electrons) are free to move around inside the conductor, <u>but</u> they cannot leave. (confined)

$\Rightarrow$ Charge builds up at ends:



$\Rightarrow \vec{E}$ field

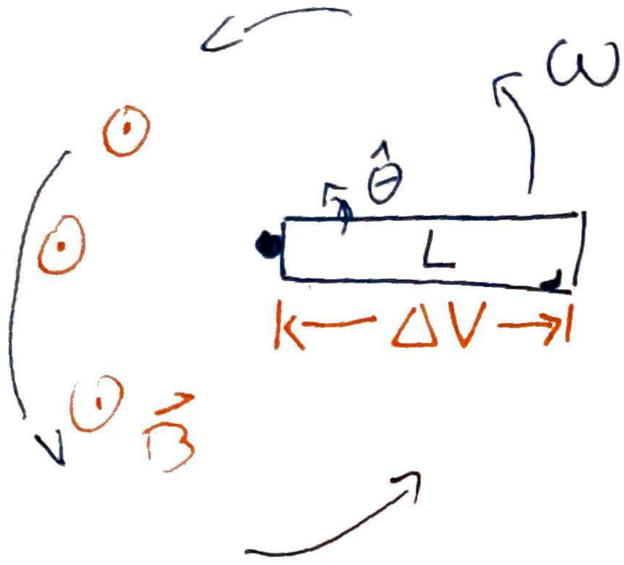
Until

$$\boxed{\vec{E} + \vec{v} \times \vec{B} = 0}$$

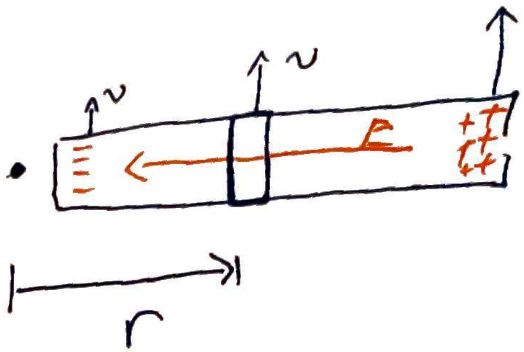
$\hookrightarrow \vec{F}_{net} = q(\vec{E} + \vec{v} \times \vec{B}) = 0$

$\Rightarrow$ $\boxed{\text{Charges move around until } \vec{F}_{net} = 0}$

4]

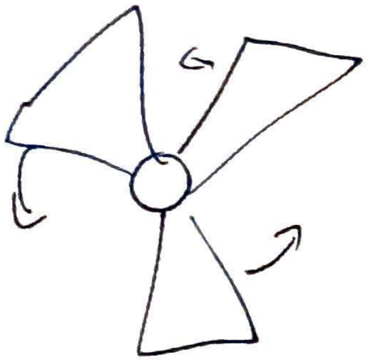


Q: What is ΔV? ($\mathcal{E}$?)



$$\boxed{|v| = \omega r}$$

$$\hat{v} = \hat{\theta} = \hat{z} \times \hat{r}$$

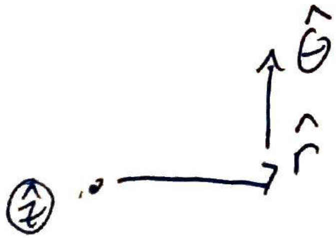


Spatially varying $\vec{E}$-field

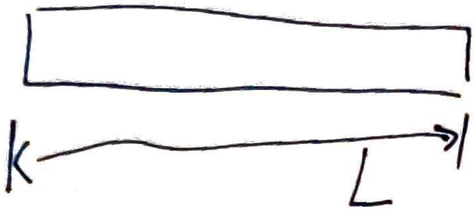
$$\vec{E} = -\vec{v} \times \vec{B}$$

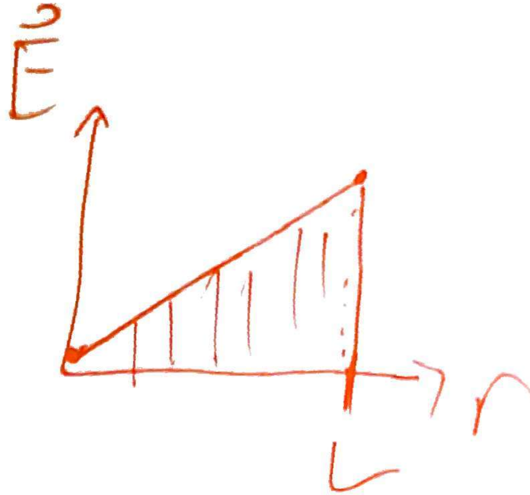
$$= -(\omega r \hat{\theta}) \times (B \hat{z})$$

$$= -\omega B r \hat{r} \quad (= -\omega B \vec{r})$$



$$\vec{E} = -\omega B r \hat{r}$$



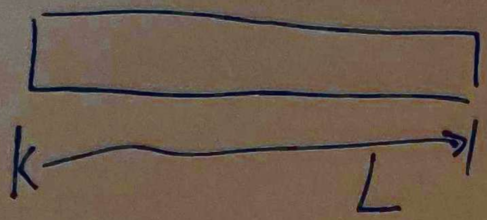


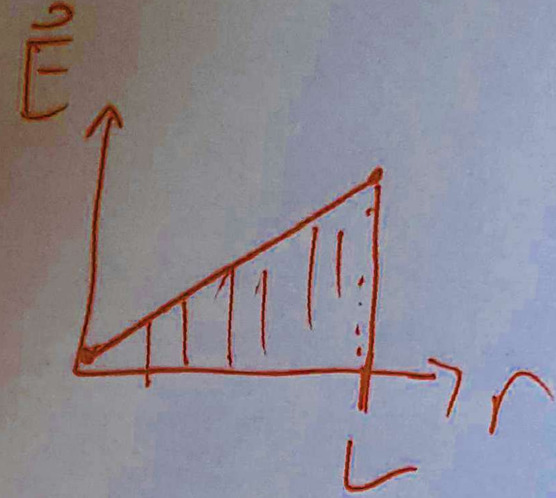
$$V(L) - V(0) = -\int_0^L \vec{E}\cdot d\vec{r}$$

$$= \int_0^L \omega B r \, d\vec{r}$$

$$\boxed{\Delta V = \frac{1}{2}\omega B L^2}$$

$$\longleftarrow \vec{E} = -\omega B r \hat{r}$$





$$V(L) - V(0) = -\int_0^L \vec{E}\cdot d\vec{r}$$

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