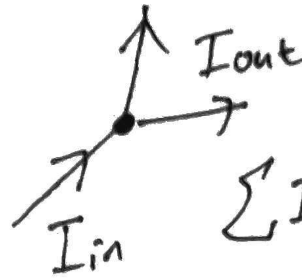
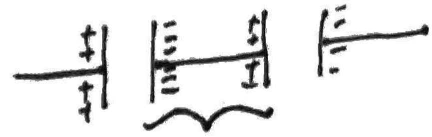
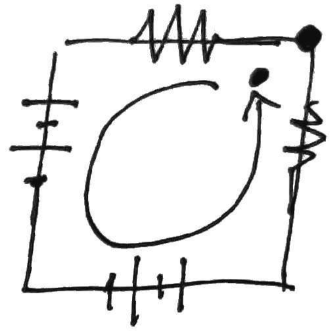
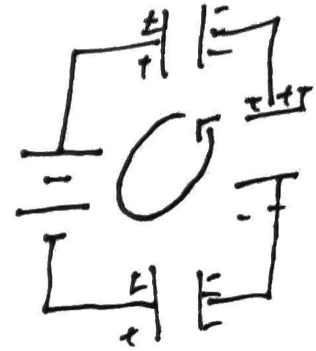
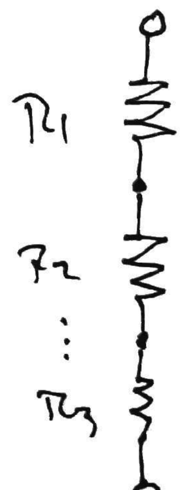
DC Circuits II - Capacitors

	Resistors	Capacitors
Conservation of charge	 $\sum I_{in} = \sum I_{out}$	 $Q_{tot} = 0$
Electric potential is well-defined	 $\sum_{loop} \Delta V = 0$	

Series:



$\cong$

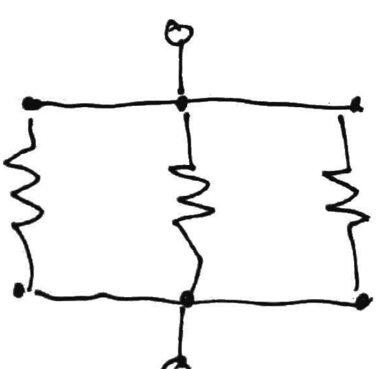
$R_{eff} = \sum_i R_i$

$\cong$

$C_{eff}^{-1} = \sum C_i^{-1}$

$\left(C = \varepsilon_0 \frac{A}{d}\right)$

Parallel:

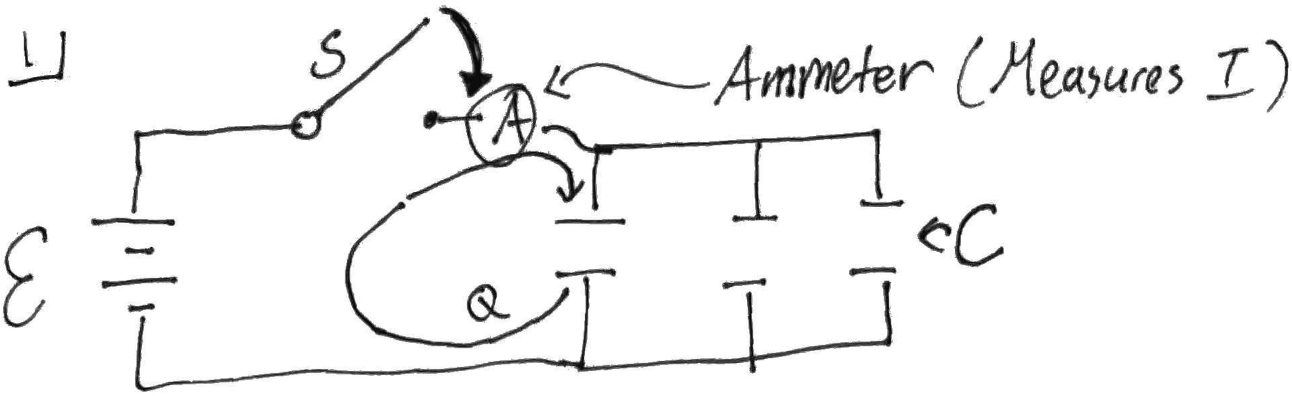


$\cong$

$R_{eff}^{-1} = \sum_i R_i^{-1}$

$C_{eff} = \sum C_i$

1

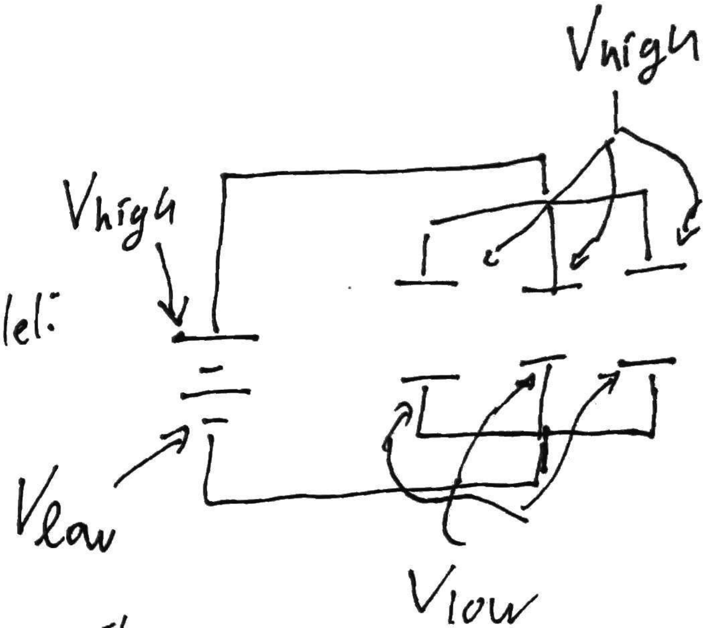


Initially, S is open, and capacitors are uncharged.

Q: How much Q_{tot} flows through (A) when switch is closed?

Two approaches:

(i) Because they are in parallel:



$\Rightarrow$ ΔV for each capacitor is the same: $\Delta V = V_{high} - V_{low} = \mathcal{E}$

Each capacitor ends up w/ $Q = C \cdot \Delta V$
$= C\mathcal{E}$

$$\boxed{Q_{tot} = Q + Q + Q = 3C\mathcal{E}}$$

$\Delta V_1 = Q_1' / C_1$, $\Delta V_2 = Q_2' / C_2$

$\hookrightarrow \quad \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} \Rightarrow \frac{Q_1'}{Q_2'} = \frac{C_1}{C_2} = \frac{1}{3}$

$\boxed{Q_2' = 3Q_1'}$

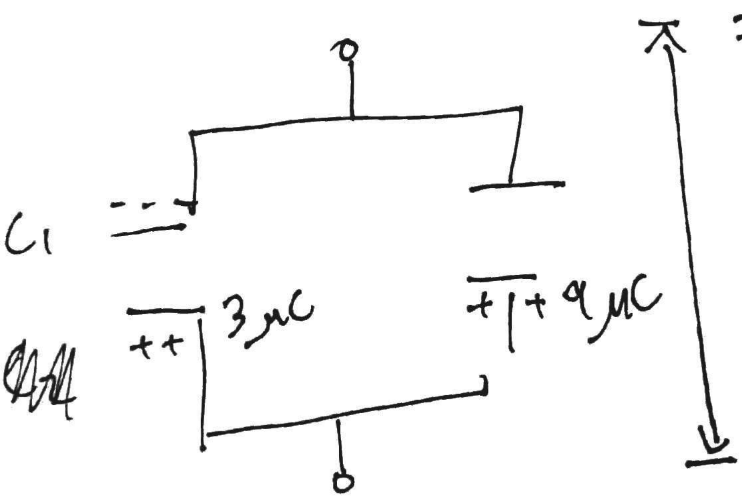
$Q_1' + Q_2' = 12\,\mu C$

$Q_1' + 3Q_1' = 4Q_1' = 12\,\mu C$

$\boxed{Q_1' = 3\,\mu C} \Rightarrow \boxed{Q_2' = 9\,\mu C}$

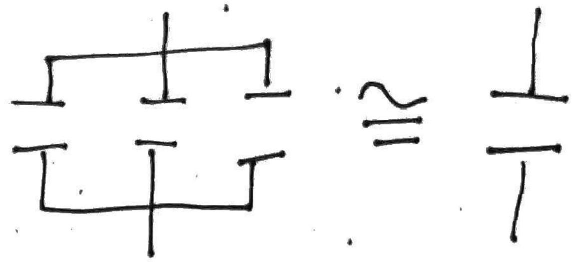
$\Delta V = \frac{Q_1'}{C_1} = \frac{3\,\mu C}{1\,\mu F} = 3\,V$

$= \frac{Q_2'}{C_2} = \frac{9\,\mu C}{3\,\mu F} = 3\,V$



$\Delta V = 3\,V$

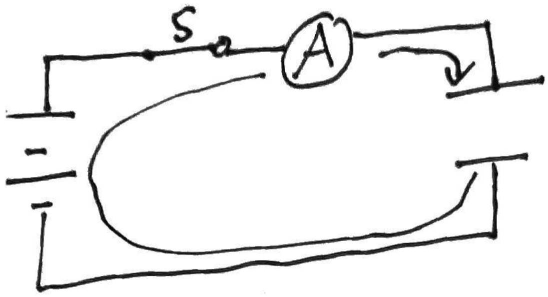
(1.1) Effective capacitor:



(Parallel)

$C_{eff} = C + C + C$

$= 3C$

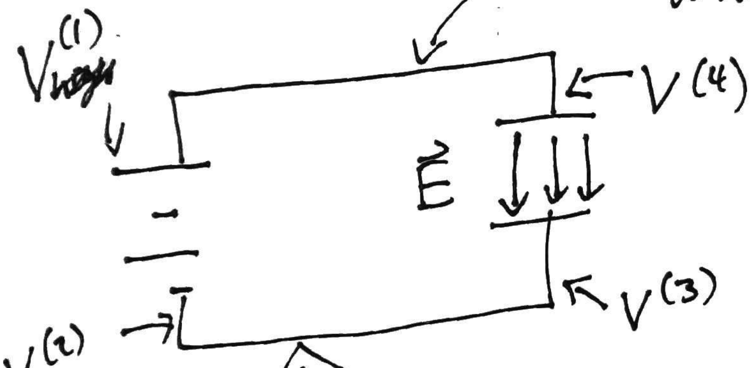


$\Delta V = \mathcal{E}$

$Q_{tot} = (3C) \cdot \mathcal{E}$ ✓

Why does the C get charged?

When I close the switch,



↳ Connect top of battery, to top of capacitor w/ wire

$\Rightarrow V^{(1)} = V^{(4)}$

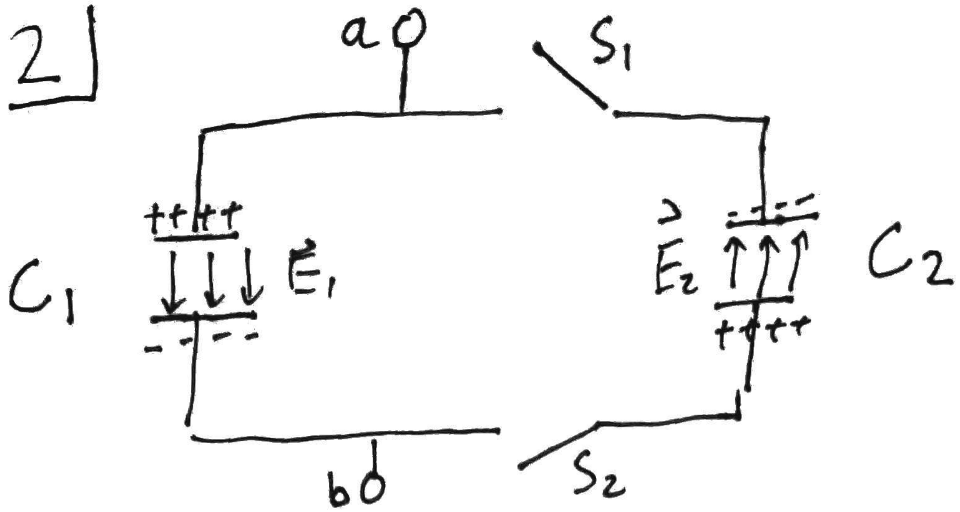
$V^{(2)} = V^{(3)}$

$\Delta V_{capacitor} \equiv V^{(4)} - V^{(3)} = V^{(1)} - V^{(2)}$

$\equiv \Delta V_{battery} = \mathcal{E} \neq 0$

↳ Charge $Q = C \cdot \Delta V$

2)



($\vec{E}$ points from + to − charges, from $V_{high} \to V_{low}$)

Initially, I have two charged capacitors, s.t. $|\Delta V_1| = |\Delta V_2| = 6\ V$

$C_1 = 1\ \mu F$, $C_2 = 3\ \mu F$

(a) Initial charges?

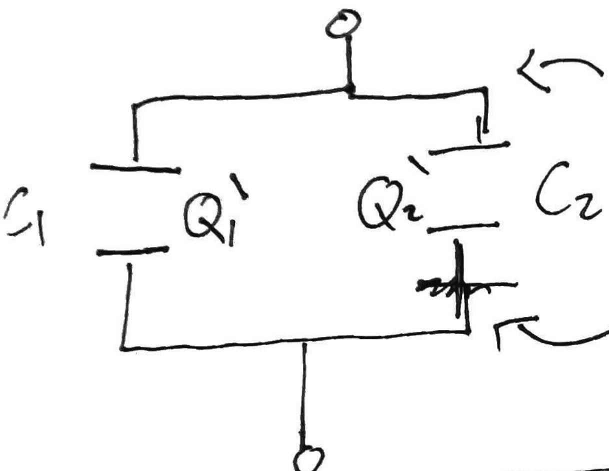
$Q_1 = C_1 \cdot \Delta V_1 = 1\ \mu F \cdot 6\ V = 6\ \mu C$

$Q_2 = C_2 \cdot \Delta V_2 = 18\ \mu C$

(b) What happens when switches are closed?

(If no external Voltage applied

↳ total charges on top & bottom halves each conserved)



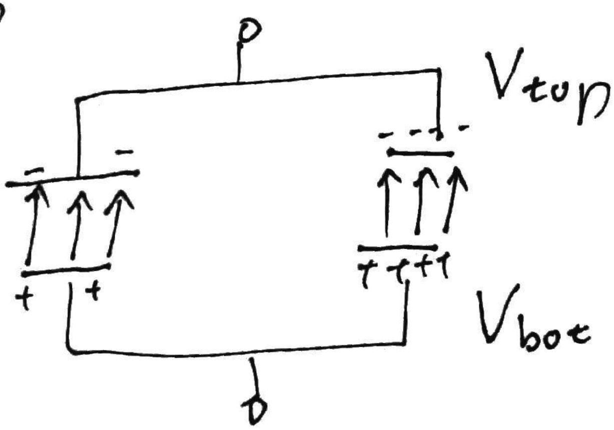
$Q_{top} = +6\mu C - 18\mu C = -12\mu C$

$Q_{bot} = -6\mu C + 18\mu C = +12\mu C$

$$Q_1' + Q_2' = 12\mu C$$

$\Delta V_1 = \Delta V_2 = V_{top} - V_{bot}$

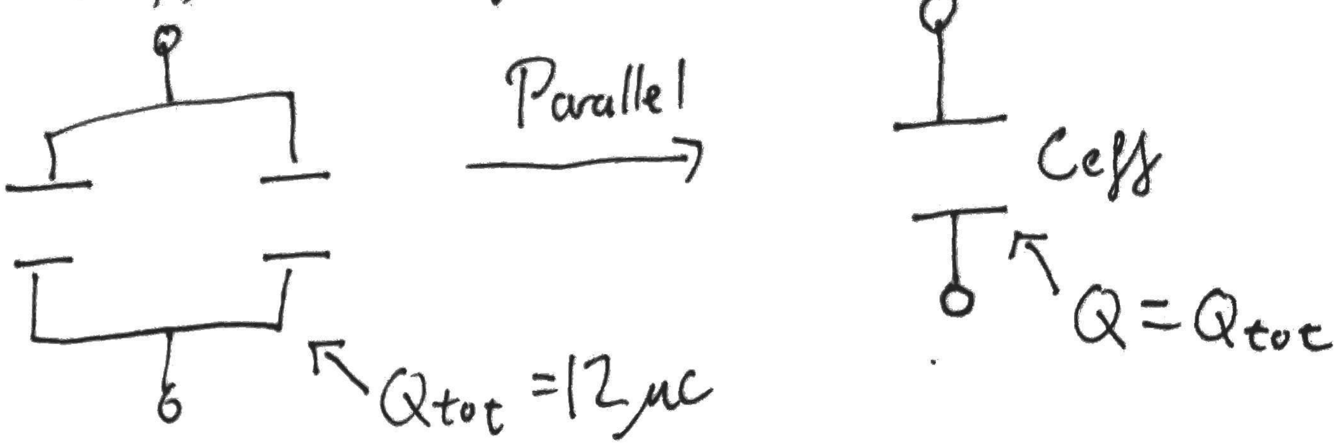
~~Each~~ Two capacitors ~~is~~ are charged in <u>same</u> direction:



↑

$\vec{E}$ in each C points in same direction (up)

(ii) Effective capacitor



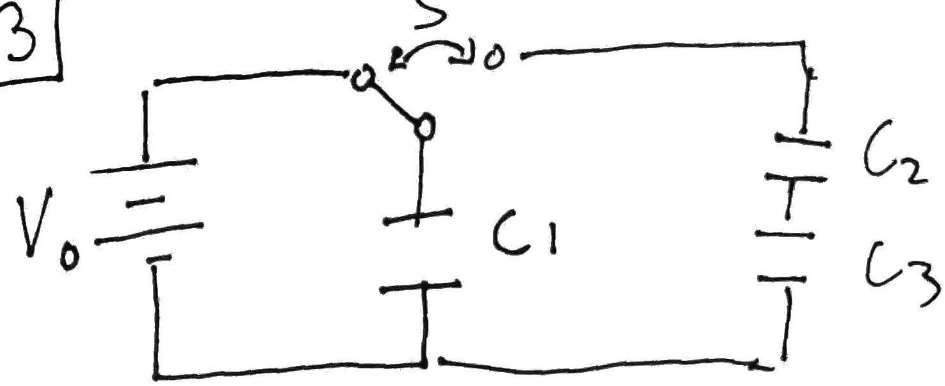
$$C_{eff} = 3\,\mu F + 1\,\mu F = 4\,\mu F$$

$$\Delta V = \frac{Q_{tot}}{C_{eff}} = \frac{12\,\mu C}{4\,\mu F} = 3\,V$$

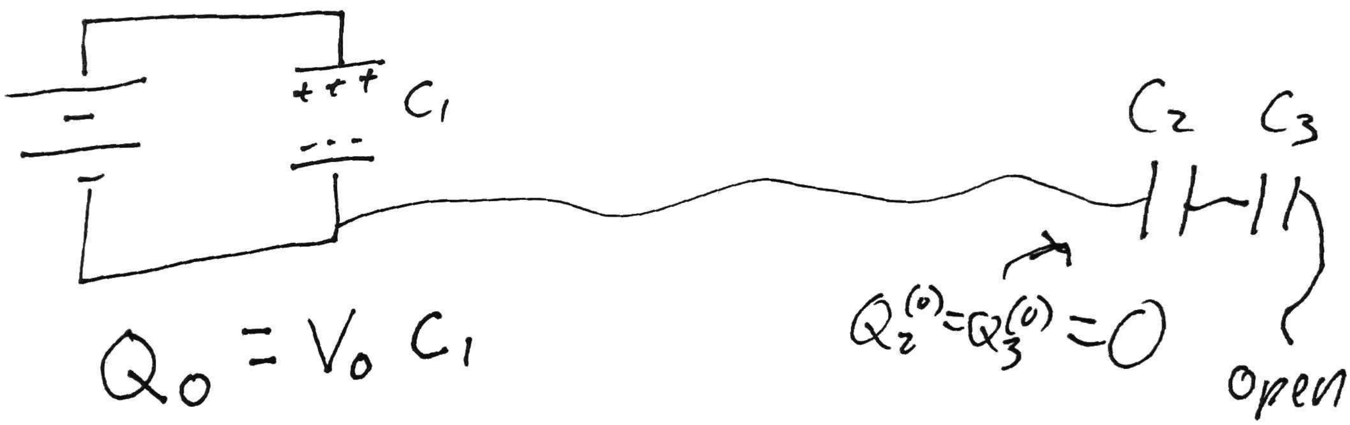
$$\Rightarrow \quad Q_1' = C_1 \cdot \Delta V = 1\,\mu F \cdot 3\,V = 3\,\mu C$$

$$Q_2' = C_2 \cdot \Delta V = 3\,\mu F \cdot 3\,V = 9\,\mu C$$

3]



Initially, switch is to the left, and only C_1 is charged:



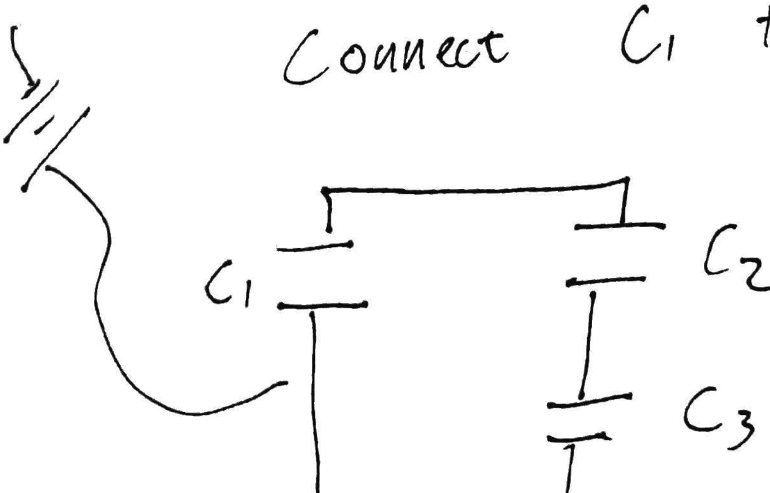
$$Q_0 = V_0 C_1$$

$$Q_2^{(0)} = Q_3^{(0)} = 0$$

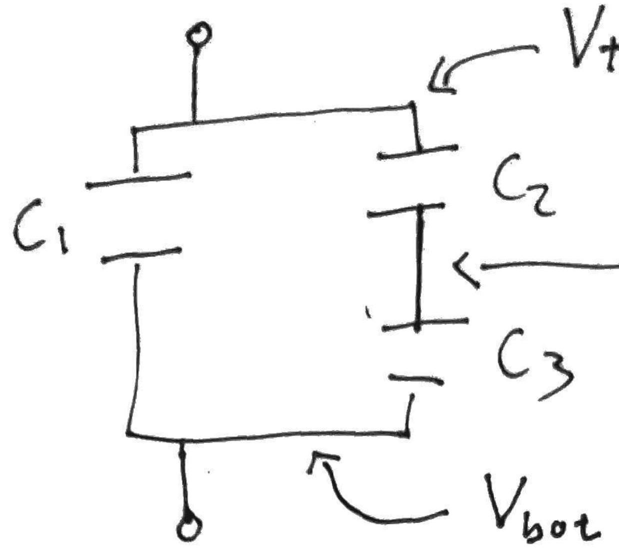
Throw switch to right

⇒ Disconnect C_1 from battery,

Connect C_1 to C_2, C_3



How does Q_0 redistribute onto other capacitors?

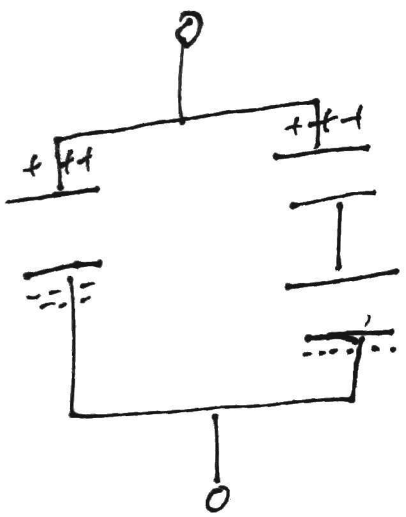


$$\Delta V_1 = V_{top} - V_{bot}$$
$$\Delta V_2 = V_{top} - V_{mid}$$
$$\Delta V_3 = V_{mid} - V_{bot}$$

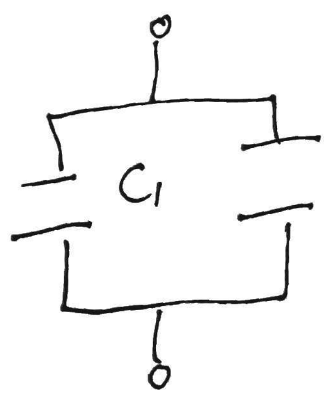
$$\Delta V_1 = \Delta V_2 + \Delta V_3$$

↳ C_1 is in parallel w/ C_2 & C_3

C_2 & C_3 are in series.



$\cong$



$$C_{23} = \left(\frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$

$\cong$

$$C_{eff} = C_1 + C_{23}$$

$$Q_{tot} = Q_0$$

Because →

the $\pm Q_0$ can only move along wires (can't hop across)

$$Q_2 = Q_0 - Q_1 = Q_0\left(1 - \frac{C_1}{C_1 + C_{23}}\right)$$

$$= Q_0 \frac{C_{23}}{C_1 + C_{23}}$$

$$Q_3 = Q_0 - Q_1 = Q_0 \frac{C_{23}}{C_1 + C_{23}}$$

Voltage drops:

$$\Delta V_2 = Q_2 / C_2 = Q_0 \frac{C_{23}/C_2}{C_1 + C_{23}}$$

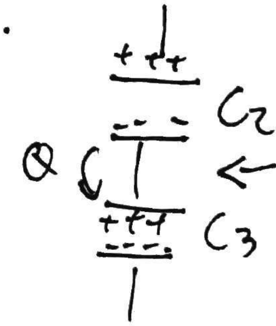
$$\Delta V_3 = Q_3 / C_3 = Q_0 \frac{C_{23}/C_3}{C_1 + C_{23}}$$

(check: $\Delta V_2 + \Delta V_3 = \Delta V_1$)

($C_2 > C_3 \Rightarrow \Delta V_2 < \Delta V_3$, $\Delta V_2 = \frac{C_3}{C_2} \cdot \Delta V_3$)

Found $Q_2 = Q_3$.

Can we explain this?



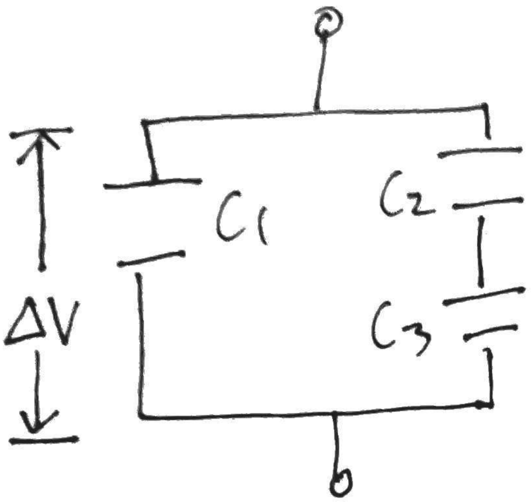
Middle region initially uncharged.

$$0 = -Q_2 + Q_3$$

$$\Rightarrow \boxed{Q_2 = Q_3}$$

$$\Delta V_{eff} = \frac{Q_{tot}}{C_{eff}} = \frac{Q_0}{C_1 + C_{23}} = \frac{V_0 C_1}{C_1 + C_{23}}$$

$$= V_0 \cdot \frac{C_1}{C_1 + C_{23}}$$



$$\Delta V_1 = \Delta V_{eff} = V_0 \cdot \underbrace{\frac{C_1}{C_1 + C_{23}}}_{\leq 1}$$

$$Q_1 = C_1 \cdot \Delta V_1 = (C_1 V_0) \cdot \frac{C_1}{C_1 + C_{23}} = Q_0 \cdot \frac{C_1}{C_1 + C_{23}}$$

Charge conservation: $Q_0 = Q_1 + Q_2$ (+ charges on top)

$Q_0 = Q_1 + Q_3$ (− charges on bottom)