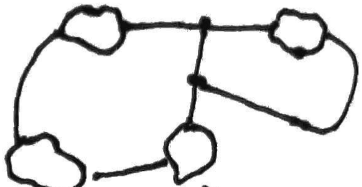
DC Circuits - Resistors and Batteries

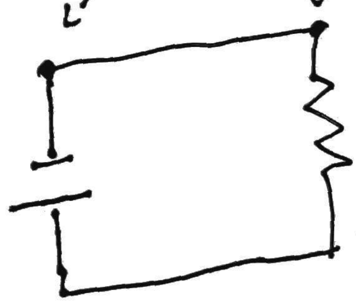
Steady state, "lumped element" approximation

(RC Circuits are ~~not~~ steady)



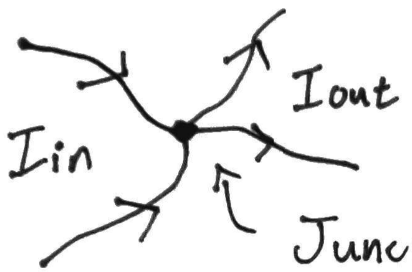
B, R, C, ...

"0th Rule": Two points connected by a wire are at the same potential:



Kirchoff's Laws — Conservation laws for circuits

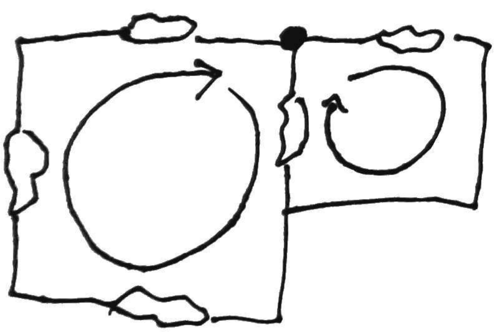
1. "Junction rule" / (Conservation of charge)



$$\boxed{\sum I_{in} = \sum I_{out}}$$

Junction — multiple wires attached at a point

2. "Loop rule" / (Conservation of energy, Electric potential is well-defined)



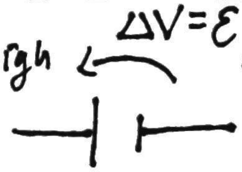
Loop: <u>Directed</u> path along some part of a circuit that ends where it starts

↳ $V_{start} = V_{end}$

$$\Rightarrow \boxed{\sum_{\substack{\text{elements} \\ \text{in loop}}} \Delta V = 0}$$

$V_{high} - V_{low}$
$\parallel$
$V = IR$

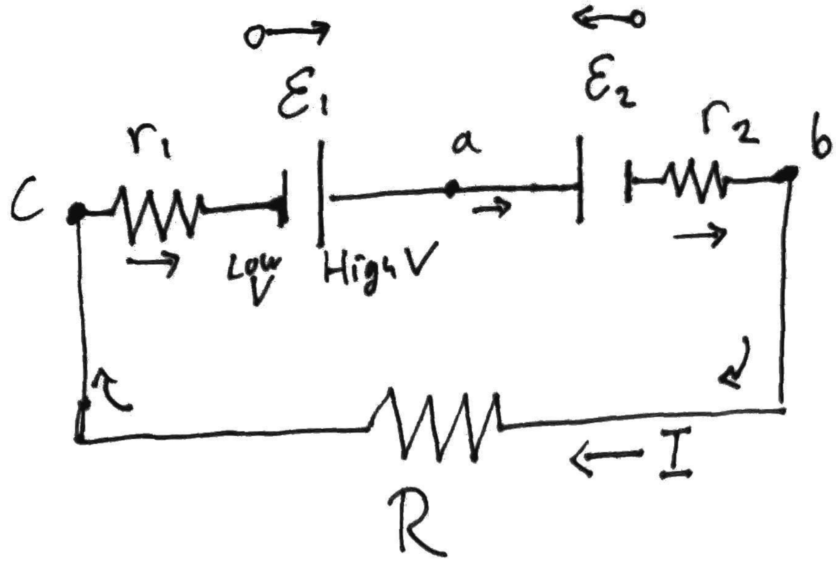
Basic elements:

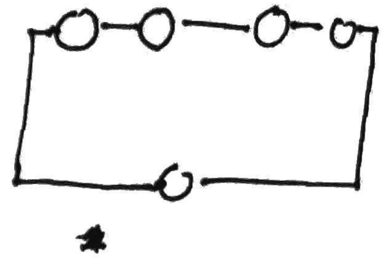


Battery

Resistors

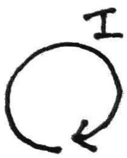
1]





(a) Constant current I, because no branches.

All junctions are like: $\xrightarrow{I_{in}} O \xrightarrow{} I_{out}$ ← 1st Law



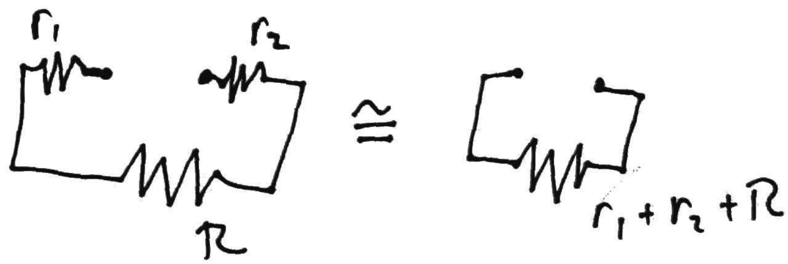
Batteries: $+\varepsilon_1$, $-\varepsilon_2$

Resistors: $-Ir_2$, $-IR$, $-Ir_1$

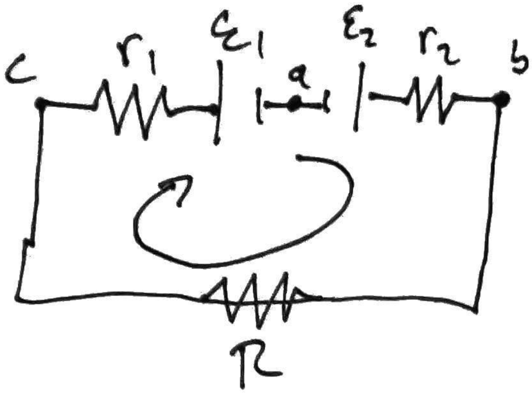
2nd Law / Loop rule: $\varepsilon_1 - \varepsilon_2 - I(r_2 + R + r_1) = 0$

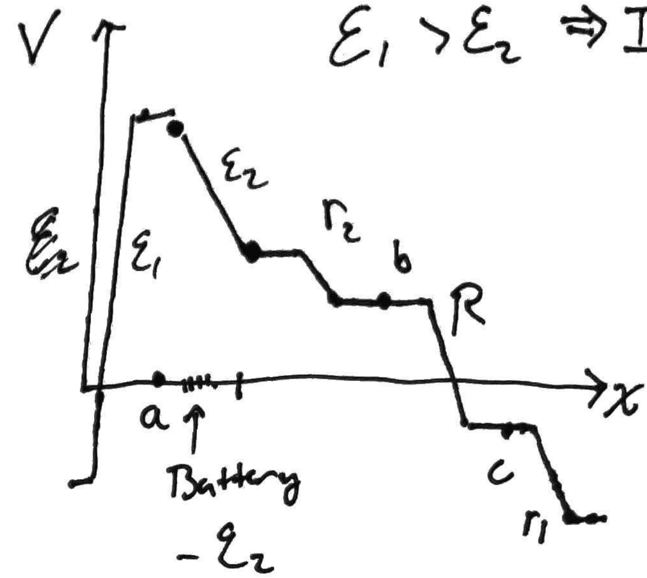
Solve for I: $I = \dfrac{\varepsilon_1 - \varepsilon_2}{r_1 + r_2 + R}$

(b) Alternatively:



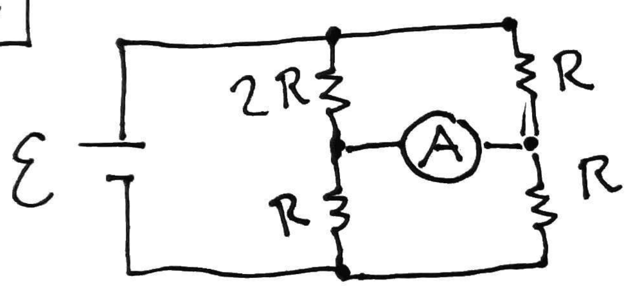
(c) Plot of potential along loop:





$\mathcal{E}_1 > \mathcal{E}_2 \Rightarrow I > 0$

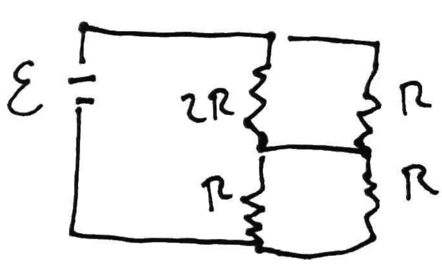
3)



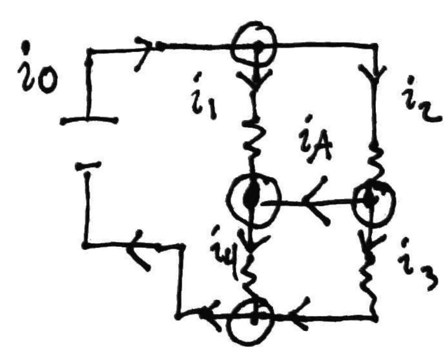
In terms of $\mathcal{E}$, R want to find the current flowing through (A)

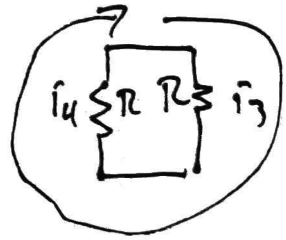
(A has 0 resistance)

≡



• First, assign current directions.

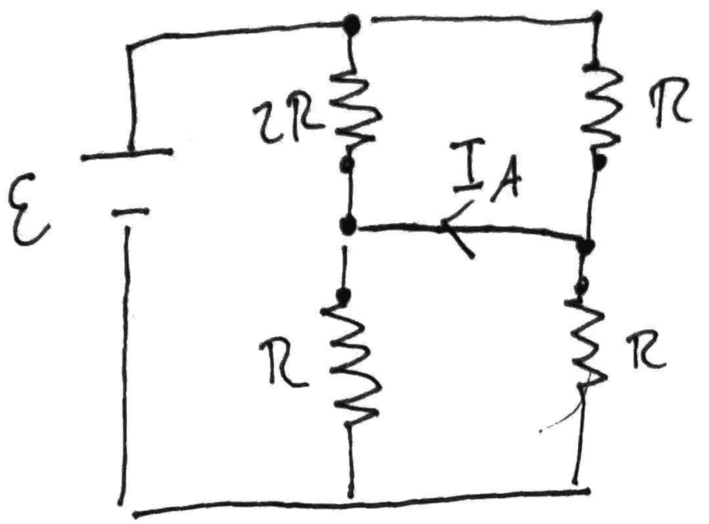




$\Rightarrow -R\, i_3 + R\, i_4 = 0$

$i_4 - i_3 = 0$

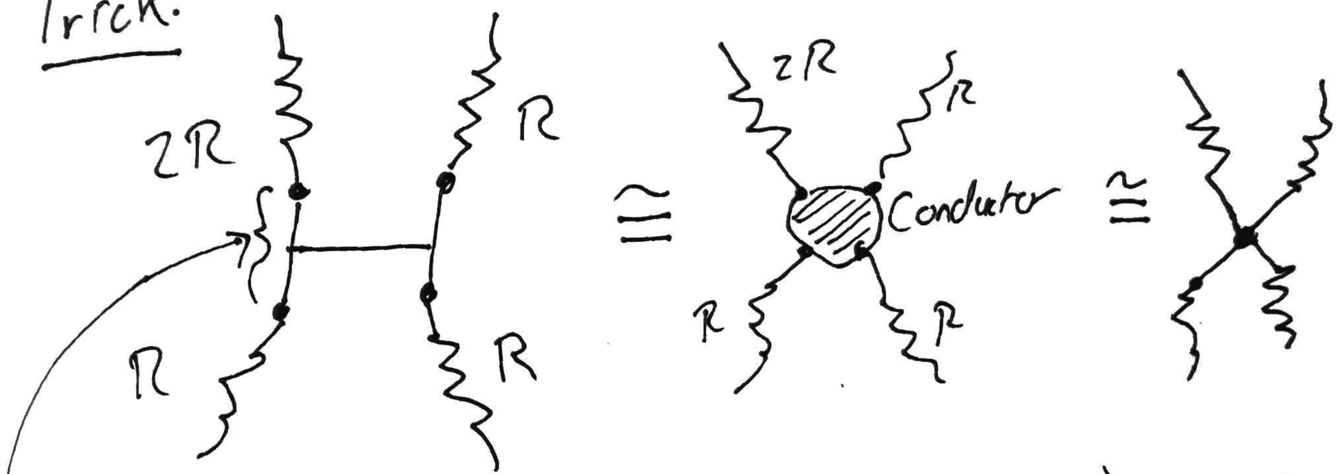
3]



Goal: Find I_A, given $\mathcal{E}$, R

Could apply Kirchoff's laws. (3 loops, 6 junctions...)

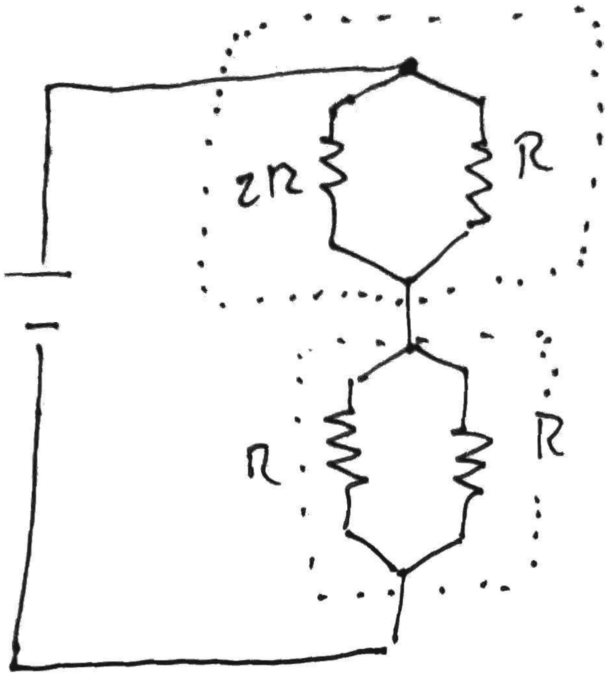
<u>Trick:</u>



Because connected by ideal wires, everything in this region is at <u>same potential</u>.

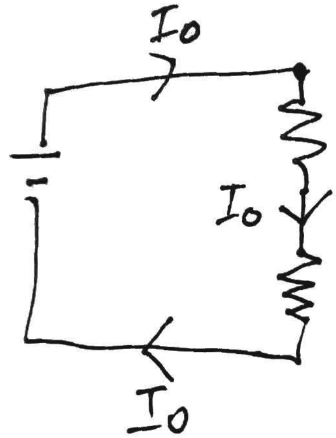
Equivalent as far as voltage differences are concerned.

Parallel:

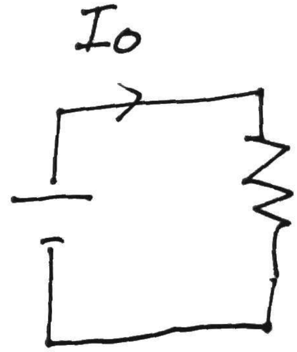


$$R_1 = \left(\frac{1}{2R} + \frac{1}{R}\right)^{-1} = \frac{2}{3}R$$

$$R_2 = \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = \frac{1}{2}R$$



Series $\simeq$



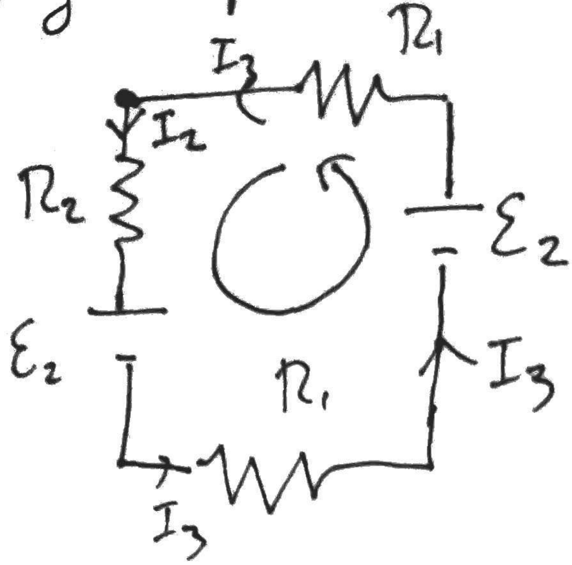
$$R_{eff} = R_1 + R_2 = \frac{7}{6}R$$

$$\boxed{I_0 = \frac{\mathcal{E}}{R_{eff}} = \frac{6}{7}\,\mathcal{E}/R}$$

$\hookrightarrow$ $V_1 = I_0 R_1 = \frac{6}{7}\frac{\mathcal{E}}{R} \cdot \frac{2}{3}R = \frac{4}{7}\mathcal{E}$

$V_2 = I_0 R_1 = \frac{6}{7}\frac{\mathcal{E}}{R} \cdot \frac{1}{2}R = \frac{3}{7}\mathcal{E}$

• Right loop:



$$0 = -I_2 R_2 - \mathcal{E}_2$$
$$- I_3 R_1 + \mathcal{E}_2 - I_3 R_1$$
$$= -I_2 R_2 - 2 I_3 R_1$$
$$= -I_2 (4\Omega) - 2 I_3 (2\Omega)$$

$$\Rightarrow 0 = -(I_2 + I_3)$$

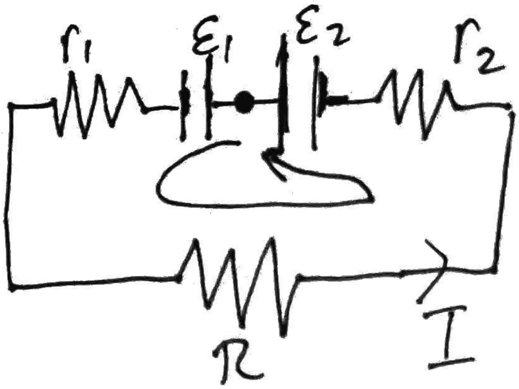
$$\Rightarrow I_2 = -I_3 \quad \text{(Right loop)}$$

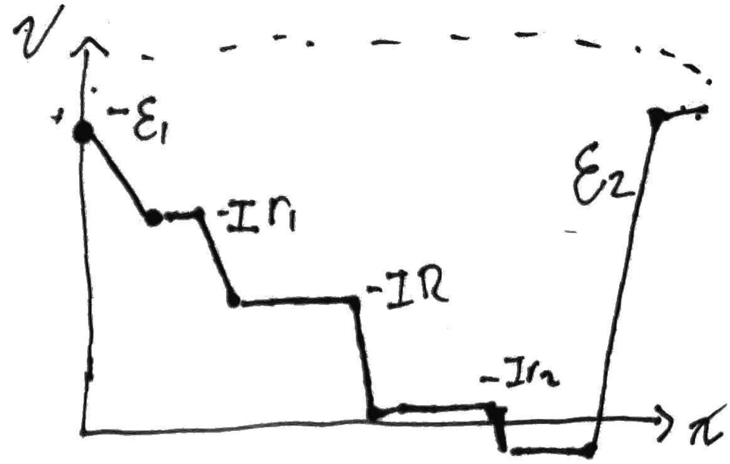
$$I_1 - I_2 = 1\,A \quad \text{(Left loop)}$$

$$I_1 + I_2 = I_3$$

$$I_2 = -I_3 = -(2I_2 + 1) \Rightarrow \boxed{I_2 = -1/3}\ A$$

$$\boxed{\begin{aligned} I_1 &= 1 + I_2 = 2/3\ A \\ I_3 &= -I_2 = +1/3\ A \end{aligned}}$$





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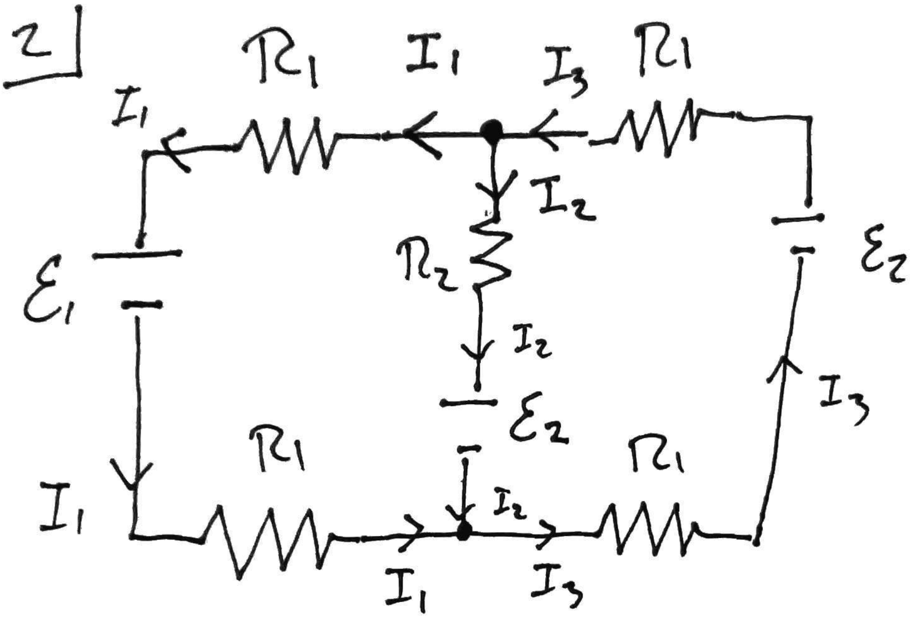


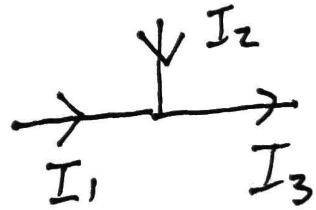
$\mathcal{E}_1 = 2\ V$

$\mathcal{E}_2 = 6\ V$

$R_1 = 2\ \Omega$

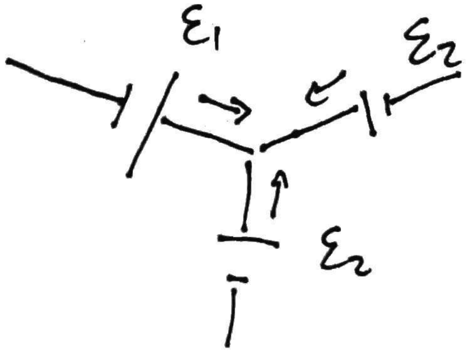
$R_2 = 4\ \Omega$

Junction rule:

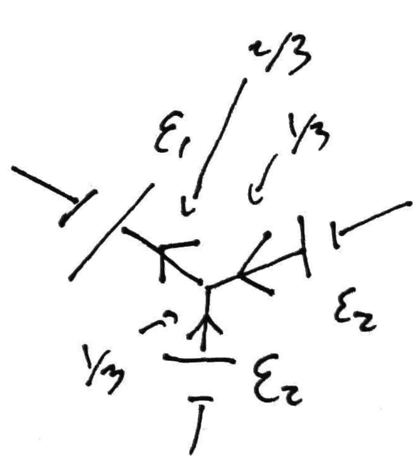


$I_1 + I_2 = I_3$

what they want:



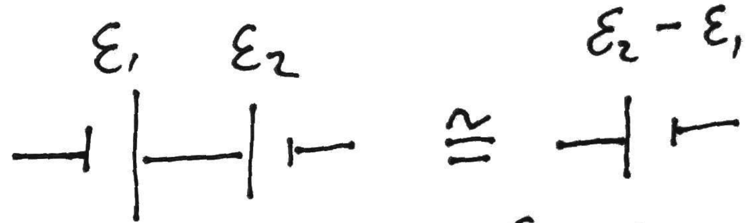
what they get:



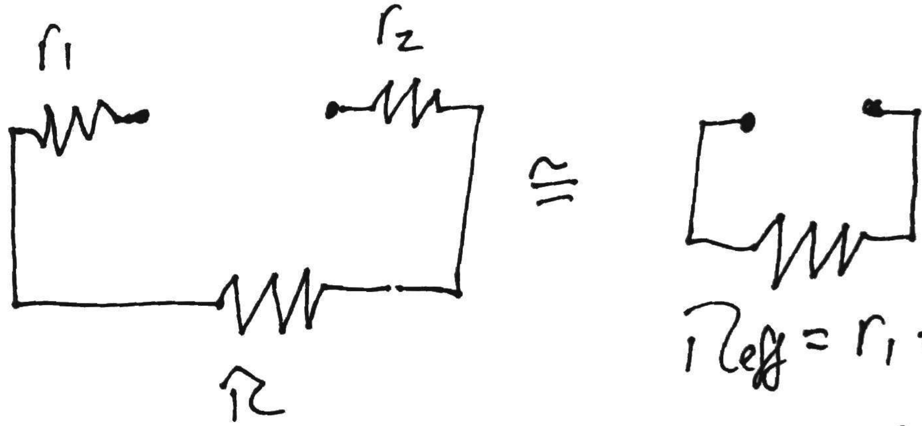
$$-I(r_1 + R + r_2) + \mathcal{E}_2 - \mathcal{E}_1 = 0$$

$$\Rightarrow \boxed{I = \frac{\mathcal{E}_2 - \mathcal{E}_1}{r_1 + R + r_2}}$$

Alternatively:



(Batteries in series)

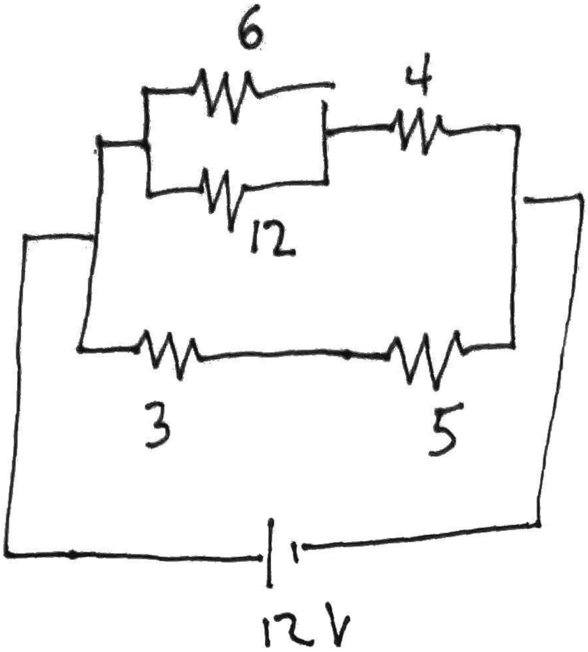


$R_{eff} = r_1 + R + r_2$

(Resistors in series)

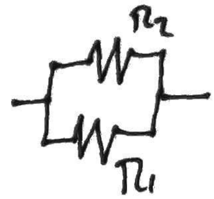
$$\Rightarrow I = \frac{\mathcal{E}_{eff}}{R_{eff}} = \frac{\mathcal{E}_2 - \mathcal{E}_1}{r_1 + R + r_2}$$

7



Find ΔV across the 5Ω resistor.

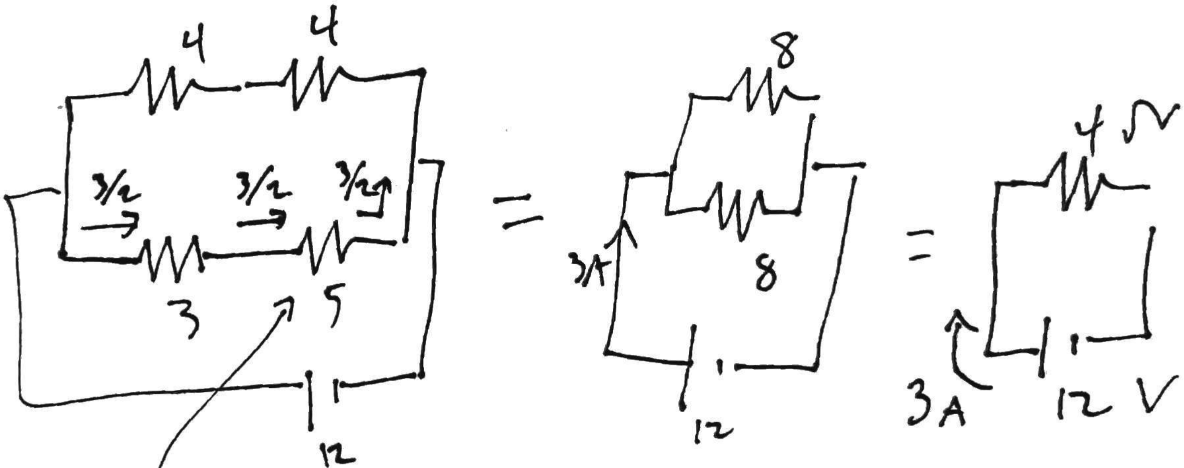
(Use series and parallel rules:



$= R_{eq}^{-1} = R_1^{-1} + R_2^{-1}$)

$= R_{eq} = R_1 + R_2$

$$\frac{1}{6} + \frac{1}{12} = \frac{2}{12} + \frac{1}{12} = \frac{1}{4}$$



$I = 3/2$ A

$$\boxed{\begin{aligned} V &= IR \\ &= 15/2 \text{ V} \end{aligned}}$$