CHAPTER 21

TEMPERATURE

With this chapter we begin our study of thermal physics, *the branch of physics that deals with the changes in the properties of systems that occur when work is done on (or by) them and heat energy is added to (or taken from) them. For systems such as confined gases the properties involved are their pressure, volume, temperature, energy, and—as we will come to learn—entropy, a property that we will introduce in Chapter 24.*

This chapter deals with temperature, *a concept that deeply underlies all of the laws of thermodynamics. We have used this concept in earlier chapters; now we must define it precisely, as we have done for all other physical concepts that we have encountered. We also introduce the concept of an* ideal gas, *which will serve as a convenient system to use in analyzing and illustrating the laws of thermodynamics.*

21-1 TEMPERATURE AND THERMAL EQUILIBRIUM

We all have an ingrained sense of temperature and, indeed, we have used this concept freely in earlier chapters. In this chapter we wish to define temperature in a rigorous way. Just as we went beyond our sense of "push" and "pull" in defining force, we need to go beyond our sense of "hot" and "cold" in defining temperature. Before we can deal directly with temperature, however, we must first establish the concept of *thermal equilibrium,* which is concerned with the question of whether or not the temperatures of two systems are equal.

Figure 21-1*a* shows two systems *A* and *B*, which, among many possibilities, might be blocks of metal or confined gases. They are isolated from one another and from their environment, by which we mean that neither energy nor matter can enter or leave either system. For example, the systems may be surrounded by walls made of thick slabs of Styrofoam, presumed to be both rigid and impermeable. Such walls are said to be *adiabatic,* which you can think of as meaning *thermally insulating.* Changes in the measured properties of either system have no effect on the properties of the other system.

As Fig. 21-1*b* shows, we can replace the adiabatic wall that separates the two systems with one that permits the

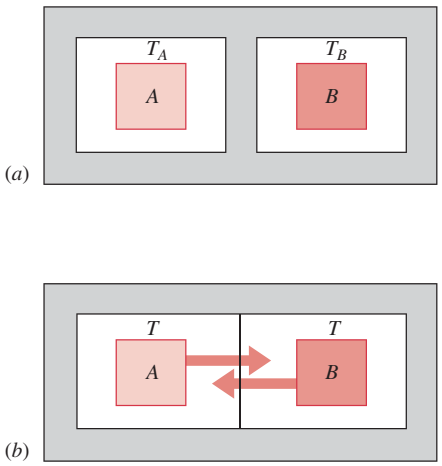

FIGURE 21-1. (*a*) Systems *A* and *B* are separated by an adiabatic wall. The systems have different temperatures T_A and T_B. (*b*) Systems *A* and *B* are separated by a diathermic wall, which permits energy to be exchanged between the systems. The systems will eventually come to thermal equilibrium, upon which they have the same temperature *T*.

flow of energy in a form that we have referred to in Chapter 13 as heat. A thin but rigid sheet of copper might be an example. Such a wall is called *diathermic,* which you can think of as *thermally conducting.*

When the two systems are placed in contact through a diathermic wall, the passage of heat energy through the wall—if it occurs—causes the properties of the two systems to change. If the systems are confined gases, for example, their pressures might change. The changes are relatively rapid at first but become slower as time goes on, until finally all measured properties of each system approach constant values. When this occurs, we say that the two systems are in *thermal equilibrium* with each other. Thus a test of whether or not two systems are in thermal equilibrium is to place them in thermal contact; if their properties do not change, they are in thermal equilibrium; if their properties do change they are not.

It might be inconvenient, or even impossible, to put two systems in thermal contact with each other through a diathermic wall. (The systems might be too bulky to move easily, or they might be very far apart.) We therefore generalize the concept of thermal equilibrium so that systems need not be brought into thermal contact with each other.

One way to test such separated systems is to use a third system C. By placing C in contact with A and then with B, we could discover whether A and B are in thermal equilibrium without ever bringing A and B into direct contact. This is summarized as a postulate called the *zeroth law of thermodynamics,* which is often stated as follows:

> *If systems A and B are each in thermal equilibrium with a third system C, then A and B are in thermal equilibrium with each other.*

This law may seem simple but it is not at all obvious. There are other situations in which a system C may have equivalent interactions with two systems A and B, but A and B do not have a similar interaction with each other. For example, if A and B are unmagnetized iron nails and C is a magnet, then A and C attract each other as do B and C. However, A and B do not.

The zeroth law came to light in the 1930s, long after the first and second laws of thermodynamics had been proposed, accepted, and named. As we discuss later, the zeroth law underlies the concept of temperature, which is fundamental to the first and second laws. The law that establishes the concept of temperature should have a lower number, hence zero.

Temperature

When two systems are in thermal equilibrium, we say that they have the same *temperature.* For example, suppose the systems are two gases that initially have different temperatures, pressures, and volumes. After we place them in contact and wait a sufficiently long time for them to reach thermal equilibrium, their pressures will in general not be equal, nor will their volumes; their temperatures, however, will always be equal in thermal equilibrium. *It is only through this argument based on thermal equilibrium that the notion of temperature can be introduced into physics.*

Although temperature in its everyday use is familiar to all of us, it is necessary to give it a precise meaning if it is to be of value as a scientific measure. Our subjective notion of temperature is not at all reliable. A familiar experience is to touch a metal railing outdoors on a very cold day and then touch a nearby wooden object. The railing well feel colder although in fact both are at the same temperature. What you are testing when you touch a cold object is not only its temperature but also its ability to transfer energy (as heat) away from your (presumably warmer) hand. In such cases your hand is giving a subjective and incorrect measure of temperature. You can also test your subjectivity convincingly by soaking your left hand in cold water and your right hand in warm water. If you then quickly put both hands in water of intermediate temperature, your left hand will sense that the water is warmer than it actually is and your right hand will sense that it is colder.

In practical use of the zeroth law, we identify system C, to which the statement of the law refers, as a *thermometer.* If the thermometer comes separately into thermal equilibrium with systems A and B (which might be widely separated buckets of water) and indicates the same reading, then we may conclude that A and B are in thermal equilibrium and thus indeed have the same temperature. Note that, to test whether two systems have the same temperature, we do not have to establish a temperature scale. If our thermometer (system C) is of the mercury-in-glass type, for example, we do not need to have it marked off in degrees. Simply put the thermometer in contact with system A, mark the mercury level, and then put it in contact with system B, noting whether the mercury reaches the same level.

A statement of the zeroth law in terms of temperature is the following:

> *There exists a scalar quantity called temperature, which is a property of all thermodynamic systems in equilibrium. Two systems are in thermal equilibrium if and only if their temperatures are equal.*

The zeroth law thus defines the concept of temperature and permits us to build and use thermometers.

21-2 TEMPERATURE SCALES

As Table 1-1 shows, temperature (symbol T) is one of the seven base units of the International System of Units (SI). As such we must define it carefully and devise procedures for measuring it that can be reproduced in laboratories around the world. Later in this section we will discuss thermometers based on the familiar Fahrenheit and Celsius scales. These, however, are scales of practical convenience and temperatures measured on them have no deep physical

meaning. The scale that is universally adopted as fundamental in physics is the *Kelvin scale.* It is based on the recognition that although there is no apparent limit to how high the temperature of a system can be, there *is* a limit to how low it can be. This *absolute zero of temperature* is defined as zero on the Kelvin scale, which measures temperatures in degrees above this absolute lower limit. Where temperature appears in any equation of fundamental importance in physics, it is certain to refer to this Kelvin (or absolute) scale.

To establish the size of the degree on the Kelvin scale we need to identify a specific calibrating system to which, by international agreement, we assign a specific temperature. We choose for this purpose an arrangement in which ice, liquid water, and water vapor coexist in thermal equilibrium. This point, which is very close to the freezing point of water at atmospheric pressure, is called the *triple point of water.* (The triple point was chosen, rather than the freezing point, because it is more consistently reproducible.) Figure 21-2 shows a triple-point cell of the type used at the National Institute of Standards and Technology (NIST). A thermometer to be calibrated is inserted into the well of the triple-point cell.

The Kelvin temperature at the triple point has been set by international agreement in 1954 to be

$$T_{tr} = 273.16\ \text{K} \qquad \text{(exactly)}, \tag{21-1}$$

where K (= kelvin) is the base unit of temperature on the Kelvin scale. The *kelvin,* which is the name we give to the degree on the Kelvin scale, is thus defined as 1/273.16 of the temperature of the triple point of water. In place of Eq. 21-1, the international community could equally well have chosen $T_{tr} = 100$ K, or any other number, but they did not. The choice they actually made was designed so that the size of the degree on the Kelvin scale (1 kelvin) would equal the size of the degree on the already well-established Celsius scale.

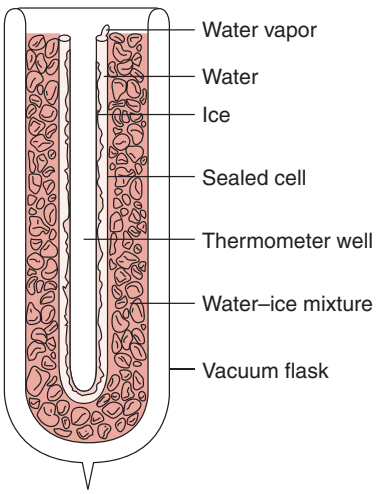

FIGURE 21-2. The National Institute of Standards and Technology triple-point cell. The U-shaped inner cell contains pure water and is sealed after all the air has been removed. It is immersed in a water–ice bath. The system is at the triple point when ice, water, and water vapor are all present, and in equilibrium, inside the cell. The thermometer to be calibrated is inserted into the central well.

Note that we do not use the degree symbol in reporting a temperature on the Kelvin scale. We might say, for example, that the melting point of lead is 600.7 K, or 600.7 kelvin.

It remains to describe how the Kelvin temperature of a system is actually measured; we shall do so in Section 21-3.

The Celsius and the Fahrenheit Temperature Scales

In nearly all the countries of the world the Celsius scale (formerly called the centigrade scale) is used for all popular and commercial—and some scientific—measurements. Historically, this scale was based on two calibration points: the normal freezing point of water, defined to be 0°C, and the normal boiling point of water, defined to be 100°C. These two points were used to calibrate thermometers and other temperatures were then deduced by interpolation or extrapolation. Note that the degree symbol (°) is used to express temperatures on the Celsius scale.

Today we no longer use these two fixed points to define the Celsius scale; instead, we define a temperature (T_C) on the Celsius scale in terms of the corresponding Kelvin temperature T, by

$$T_C = T - 273.15. \tag{21-2}$$

The freezing and boiling points of water (at a pressure of 1 atm) are now measured on the Kelvin scale and then converted to Celsius using Eq. 21-2. The experimental values are, respectively, 0.00°C and 99.975°C, in agreement (for all practical purposes) with the historical basis for defining the Celsius scale. Note also that Eq. 21-2 indicates that the Celsius temperature of the triple point of water is 0.01°C. As we pointed out earlier, this is close to the temperature of the freezing point of water. Also note that, according to Eq. 21-2, the absolute zero of temperature is −273.15°C.

The Fahrenheit scale was also based historically on two fixed points that, after several earlier choices, came to be: (1) the normal freezing point of water, which was defined to be 32°F, and (2) the normal boiling point of water, which was defined to be 212°F. The relationship between the Fahrenheit and the Celsius scales is now taken to be

$$T_F = \tfrac{9}{5}T_C + 32. \tag{21-3}$$

As for the Celsius scale, the degree symbol is used in reporting temperatures on the Fahrenheit scale, for example, 98.6°F (normal oral human body temperature).

Transferring between the Celsius and the Fahrenheit scales is easily done by remembering a few corresponding points, such as those shown in Fig. 21-3, which compares the Kelvin, Celsius, and Fahrenheit scales. It is also necessary to make use of the equality between an *interval* of 9

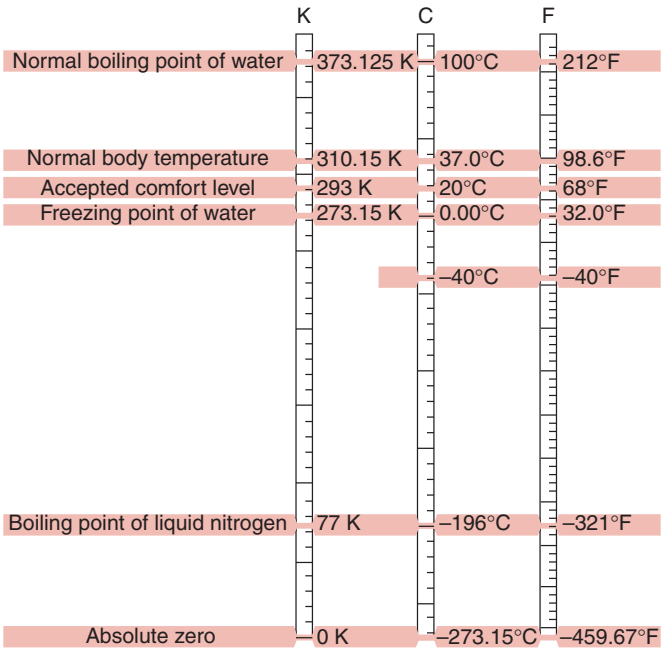

FIGURE 21-3. The Kelvin, Celsius, and Fahrenheit temperature scales compared. Note that the latter two scales coincide at − 40°.

degrees on the Fahrenheit scale and an *interval* of 5 degrees on the Celsius scale, which we express as

$$9\ \text{F}^\circ = 5\ \text{C}^\circ. \tag{21-4}$$

Note that these *intervals* are expressed as F° and C°, not as °F or °C. Thus we might write or say: "The temperature here is 90°F. It would be more pleasant if it were 15 F° cooler."

21-3 MEASURING TEMPERATURES

Here we address the problem of measuring the temperatures of a system on the Kelvin scale. Once we have made this measurement, we can easily find the temperature of the system on the Celsius and the Fahrenheit scales, using Eqs. 21-2 and 21-3. To measure a temperature we need a thermometer. What form shall it take?

In principle, any property of a substance that varies with temperature can form the basis for a thermometer. Examples might be the volume of a liquid (as in the common mercury-in-glass thermometer), the pressure of a gas kept at constant volume, the electrical resistance of a wire, the length of a strip of metal, or the color of a lamp filament, all of which vary with temperature and all of which are in common use as thermometers. *The choice of one of these properties leads to a device-sensitive or "private" temperature scale that is defined only for that property and that does not necessarily agree with other choices we might make.* Of course all thermometers will agree, by definition of Eq. 21-1, at the triple point of water. The question is, will they agree at other temperatures, either higher or lower? The answer is that they will not, as Sample Problem 21-1 shows. Even so, a "private" thermometer, when properly calibrated against accepted standards, can be useful as a secondary standard for measuring temperature. Indeed, nearly all temperature measurements are made using such secondary standard thermometers.

Let us assume that our thermometer is based on a system in which we measure the value of an as yet unspecified thermometric property X. The temperature is some function of X. We choose the simplest possible relationship—namely, a linear one

$$T^* = aX \tag{21-5}$$

in which a is a constant. We designate the temperature given by Eq. 21-5 by T^* rather than T because the temperature so measured will be a device-sensitive temperature, not a true Kelvin temperature. We can find the value of a by measuring X at the triple point of water, obtaining the value X_{tr}. We then have, for the temperature as a function of X,

$$T^*(X) = (273.16\ \text{K})\frac{X}{X_{\text{tr}}}. \tag{21-6}$$

It remains only to select a suitable temperature-dependent property X and to see whether we can establish a procedure that will yield the true Kelvin temperature rather than T^*.

SAMPLE PROBLEM 21-1. The resistance of a certain coil of platinum wire increases by a factor of 1.392 between the triple point of water and the boiling point of water at atmospheric pressure (that is, the normal boiling point). What temperature for the normal boiling point of water is measured by this thermometer?

Solution The generalized thermometric property X that appears in the defining relation of Eq. 21-6 is, in this case, the resistance R. We are not given R_{tr} but we are told that $R = 1.392\ R_{\text{tr}}$. Thus, with R substituted for X, Eq. 21-6 becomes

$$T^*(R) = T_{\text{tr}}\frac{R}{R_{\text{tr}}} = (273.16\ \text{K})(1.329) = 380.2\ \text{K}.$$

This value gives the "platinum resistance temperature" of boiling water. Other thermometers will give different values. For example, the normal boiling point of water as measured by a thermometer (a *thermocouple*) based on the electric voltage generated by two joined dissimilar wires (copper and constantan) is 412.5 K. The actual Kelvin temperature of the normal boiling point of water (see Fig. 21-3) is 373.125 K. Although such "private scale" thermometers, when properly calibrated, are indispensable for practical use, we cannot rely on them to give consistent measures of temperature on the Kelvin scale.

The Constant-Volume Gas Thermometer

The thermometric property that proves most suitable for measuring temperatures on the Kelvin scale is the pressure p exerted by a fixed volume of gas. The device for realizing

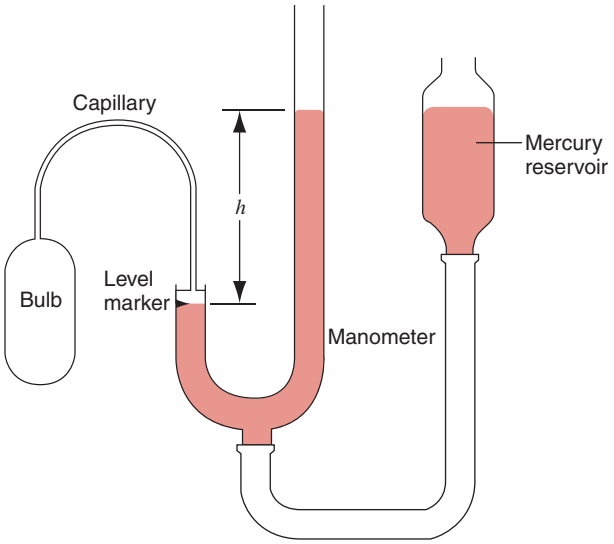

FIGURE 21-4. A constant-volume gas thermometer. The bulb can be immersed in a triple-point cell and then in the bath of a liquid whose temperature we are trying to measure. The difference between the pressure of the gas in the bulb and the atmospheric pressure is found from the height h of the mercury column in the manometer. The simplicity of this sketch greatly conceals the complexity of an actual gas thermometer such as may be found, for example, in national standardizing laboratories in many countries.

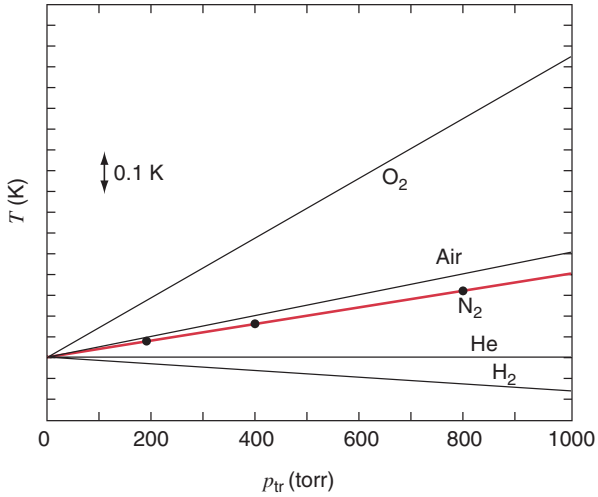

FIGURE 21-5. As the pressure of the nitrogen gas in a constant-volume gas thermometer is reduced from 800 torr to 400 and then to 200, the temperature deduced for the system approaches a limit corresponding to a pressure of zero. Other gases approach the same limit. The full range of the vertical scale is about 1 K for typical conditions.

this procedure in practice is called a *constant-volume gas thermometer.* Figure 21-4 shows a sketch of its essential features. A gas-filled bulb can be alternately immersed in a bath of the liquid whose temperature is to be measured or in a triple-point bath. The volume of the gas in the bulb, which we take to be nitrogen, is maintained constant by raising or lowering the mercury-filled reservoir, so that the level of mercury in the left arm of the manometer always coincides with a fixed marker.

The procedure for measuring a temperature is as follows:

Step 1: (*a*) Immerse the nitrogen-filled bulb in a triple-point bath and read the pressure p_{tr} of the contained gas on the manometer. Let us say that, in a particular case, $p_{tr} =$ 800 torr. (*b*) Immerse the bulb in the bath whose temperature is to be measured and read the new pressure p. Calculate T^* from Eq. 21-6, in which X is replaced with p and X_{tr} with p_{tr}. The result, which we regard as provisional, is plotted as a point at 800 torr in Fig. 21-5.

Step 2: Return the thermometer bulb to the triple-point bath and remove some of the gas, thus decreasing its density. Now p_{tr} has a smaller value—say, 400 torr. Then we return the bulb to the bath whose temperature we are trying to find, measure a new value of p, and calculate a new provisional temperature T^*, also plotted in Fig. 21-5.

We continue this procedure, reducing the amount of gas in the bulb step by step and, at each new lower value of p_{tr} calculating T^*. If we plot the values of T^* against p_{tr}, we can extrapolate the resulting curve to the intersection with the axis at $p_{tr} = 0$. The data points for nitrogen gas and the resultant straight-line extrapolation are shown in Fig. 21-5.

If we repeat this step-wise extrapolation procedure for gases other than nitrogen, we obtain results also shown in Fig. 21-5. We see that, as the triple-point pressure p_{tr} (and thus the gas density) is reduced, the temperature readings of constant-volume gas thermometers approach the same value T, no matter what gas is used. We can regard T as the temperature of the system and we define an *ideal gas temperature scale:*

$$T = (273.16\ \text{K}) \lim_{p_{tr} \to 0} \frac{p}{p_{tr}} \qquad \text{(constant } V\text{)}. \quad (21\text{-}7)$$

In this context, we define an "ideal gas" to be a gas that would read the same temperature T at all pressures, with no need for extrapolation. We will say more about the ideal gas in Section 21-5.

If temperature is to be a truly fundamental physical quantity it is absolutely necessary that its definition be independent of the properties of specific materials. It would not do, for example, to have such a basic quantity as temperature depend on the thermal expansivity of mercury, the electrical resistivity of platinum, or any other such "handbook" property. We choose the gas thermometer as our standard precisely because no such properties are involved in its operation. You can use any gas and you always get the same answer.

The lowest temperature that can be measured with a gas thermometer is about 1 K. To obtain this temperature we must use low-pressure helium, which remains a gas at lower temperatures than any other gas.

TABLE 21-1 Temperatures of Selected Systems

System	*Temperature* (K)
Plasma in fusion test reactor	10^8
Center of Sun	10^7
Surface of Sun	6×10^3
Melting point of tungsten	3.6×10^3
Freezing point of water	2.7×10^2
Normal boiling point of N_2	77
Normal boiling point of ^{4}He	4.2
Mean temperature of universe	2.7
^{3}He–^{4}He dilution refrigerator	5×10^{-3}
Adiabatic demagnetization of paramagnetic salt	10^{-3}
Bose–Einstein condensation experiments	2×10^{-8}

It can be shown that temperatures measured with the constant-volume gas thermometer are true Kelvin temperatures in the range in which the gas thermometer can be used. We must use special methods to measure Kelvin temperatures outside of this range. Table 21-1 lists the Kelvin temperatures of some systems and processes.

The International Temperature Scale

Precise measurement of a temperature with a gas thermometer is a difficult task, requiring many months of painstaking laboratory work and, when completed, has been said to be an international event. In practice therefore, the gas thermometer is used only to establish certain fixed points that can then be used to calibrate other more convenient secondary thermometers.

The International Temperature Scale has been adopted for the calibration of thermometers for scientific or industrial use. This scale consists of a set of procedures for providing in practice the best possible approximations to the Kelvin scale. The adopted scale consists of a set of fixed points, along with specific devices to be used for interpolating between these fixed points and extrapolating beyond the highest fixed point. The International Committee of Weights and Measures reviews and refines the scale about every 20 years. Table 21-2 shows the fixed points of the 1990 version of the International Temperature Scale.

TABLE 21-2 Primary Fixed Points on the 1990 International Temperature Scale[a]

Substance	*State*	*Temperature* (K)
Helium	Boiling point	3–5[c]
Hydrogen	Triple point	13.8033
Hydrogen	Boiling point[b]	17.025–17.045[c]
Hydrogen	Boiling point	20.26–20.28[c]
Neon	Triple point	24.5561
Oxygen	Triple point	54.3584
Argon	Triple point	83.8058
Mercury	Triple point	234.3156
Water	Triple point	273.16
Gallium	Melting point	302.9146
Indium	Freezing point	429.7485
Tin	Freezing point	505.078
Zinc	Freezing point	692.677
Aluminum	Freezing point	933.473
Silver	Freezing point	1234.93
Gold	Freezing point	1337.33
Copper	Freezing point	1357.77

[a] See "The International Temperature Scale of 1990 (ITS-90)," by H. Preston-Thomas, *Metrologia,* 27 (1990), p. 3.
[b] This boiling point is for a pressure of $\frac{1}{3}$ atm. All other boiling points, melting points, or freezing points are for a pressure of 1 atm.
[c] The temperature of the boiling point varies somewhat with the pressure of the gas above the liquid. The temperature scale gives the relationship between T and p that can be used to calculate T for a given p.

21-4 THERMAL EXPANSION

You can often loosen a tight metal jar lid by holding it under a stream of hot water. As its temperature rises, the metal lid expands slightly relative to the glass of the jar. Thermal expansion is not always desirable, as Fig. 21-6 suggests. Roadways of bridges usually include expansion slots to allow for changes in length of the roadway as the temperature changes.

Pipes at refineries often include an expansion loop, so that the pipe will not buckle as the temperature rises. Materials used for dental fillings have expansion properties similar to those of tooth enamel. In aircraft manufacture, rivets and other fasteners are often designed so that they are to be cooled in dry ice before insertion and then allowed to ex-

FIGURE 21-6. Railroad tracks distorted because of thermal expansion on a very hot day. Railroad tracks today come in 1500-ft lengths and, to prevent buckling, are laid at or near the maximum annual temperature of the locality.

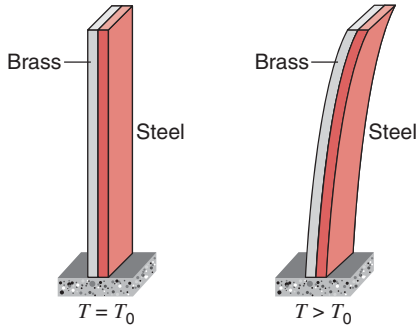

FIGURE 21-7. A bimetallic strip, consisting of a strip of brass and a strip of steel welded together, at temperature T_0. At temperatures higher than T_0, the strip bends as shown; at lower temperatures it bends the other way. Many thermostats operate on this principle, using the motion of the end of the strip to make or break an electrical contact.

pand to a tight fit. Thermometers and thermostats may be based on the differences in expansion between the components of a bimetallic strip; see Fig. 21-7. In a thermometer of a familiar type, the bimetallic strip is coiled into a helix that winds and unwinds as the temperature changes; see Fig. 21-8. The familiar liquid-in-glass thermometers are based on the fact that liquids such as mercury or alcohol expand to a different (greater) extent than do their glass containers.

We can understand this expansion by considering a simple model of the structure of a crystalline solid. The atoms are held together in a regular array by electrical forces, which are like those that would be exerted by a set of springs connecting the atoms. We can thus visualize the solid body as a microscopic bedspring (Fig. 21-9). These "springs" are quite stiff and not at all ideal (see Problem 1

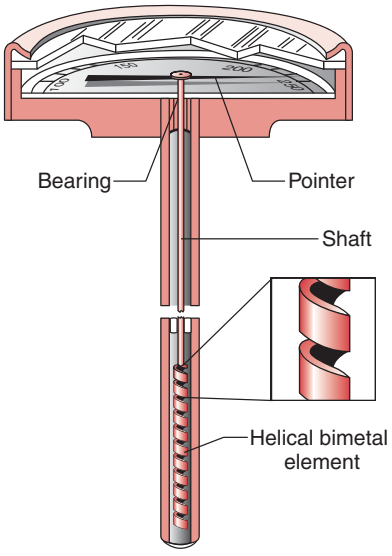

FIGURE 21-8. A thermometer based on a bimetallic strip. The strip is formed into a helix, which coils or uncoils as the temperature is changed.

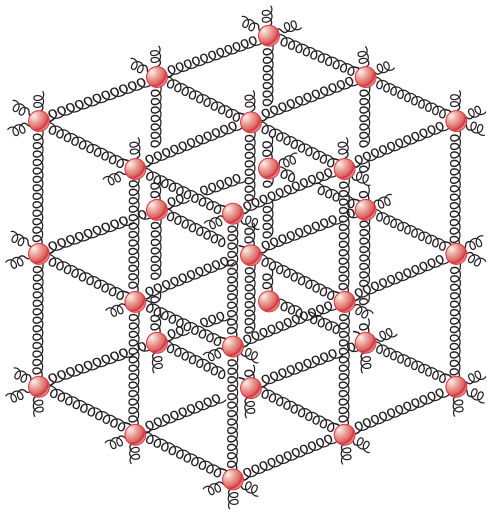

FIGURE 21-9. A solid behaves in many ways as if it were a collection of atoms joined by elastic forces (here represented by springs).

of Chapter 17), and there are about 10^{23} of them per cubic centimeter. At any temperature the atoms of the solid are vibrating. The amplitude of vibration is about 10^{-9} cm, about one-tenth of an atomic diameter, and the frequency is about 10^{13} Hz. When the temperature is increased, the atoms vibrate at larger amplitude, and the average distance between atoms increases. (See the discussion of the microscopic basis of thermal expansion at the end of this section.) This leads to an expansion of the whole solid body.

The change in *any* linear dimension of the solid, such as its length, width, or thickness, is called a *linear expansion.* If the length of this linear dimension is L, the change in temperature ΔT causes a change in length ΔL. We find from experiment that, if ΔT is small enough, this change in length ΔL is proportional to the temperature change ΔT and to the original length L. Hence we can write

$$\Delta L = \alpha L \, \Delta T, \tag{21-8}$$

where α, called the *coefficient of linear expansion,* has different values for different materials. Rewriting this formula, we obtain

$$\alpha = \frac{\Delta L/L}{\Delta T}, \tag{21-9}$$

so that α has the meaning of a fractional change in length per degree temperature change.

Strictly speaking, the value of α depends on the actual temperature and the reference temperature chosen to determine L (see Problem 5). However, its variation is usually negligible compared to the accuracy with which measurements need to be made. It is often sufficient to choose an average value that can be treated as a constant over a certain temperature range. In Table 21-3 we list the experimental values for the average coefficient of linear expansion of several common solids. For all the substances listed, the

TABLE 21-3 Some Average Coefficients of Linear Expansion[a]

Substance	$\alpha(10^{-6}$ per C°)
Ice	51
Lead	29
Aluminum	23
Brass	19
Copper	17
Steel	11
Glass (ordinary)	9
Glass (Pyrex)	3.2
Invar alloy	0.7
Quartz (fused)	0.5

[a] Typical average values in the temperature range 0°C to 100°C are shown, except for ice in which the range is − 10°C to 0°C.

change in size consists of an expansion as the temperature rises, because α is positive. The order of magnitude of the expansion is about 1 millimeter per meter length per 100 Celsius degrees. (Note the use of C°, not °C, to express temperature changes here. Note also, that since 1 K is the same as 1 C°, we can use either Kelvin or Celsius temperature differences in Eq. 21-9.)

SAMPLE PROBLEM 21-2. A steel metric scale is to be ruled so that the millimeter intervals are accurate to within about 5×10^{-5} mm at a certain temperature. What is the maximum temperature variation allowable during the ruling?

Solution From Eq. 21-8, we have

$$\Delta T = \frac{\Delta L}{\alpha L} = \frac{5 \times 10^{-5}\ \text{mm}}{(11 \times 10^{-6}/\text{C°})(1.0\ \text{mm})} = 4.5\ \text{C°},$$

where we have used the value of α for steel from Table 21-3. The temperature during the ruling must be kept constant to within about 5 C°, and the scale must be used within that same interval of temperature at which it was ruled.

Note that if the alloy invar were used instead of steel, we could achieve the same precision over a temperature interval of about 75 C°; or, equivalently, if we could maintain the same temperature variation (5 C°), we could achieve an accuracy due to temperature changes of about 3×10^{-6} mm.

For many solids, called *isotropic,* the percent change in length for a given temperature change is the same for all lines in the solid. The expansion is quite analogous to a photographic enlargement, except that a solid is three-dimensional. Thus, if you have a flat plate with a hole punched in it, $\Delta L/L$ $(= \alpha\, \Delta T)$ for a given ΔT is the same for the length, thickness, face diagonal, body diagonal, and hole diameter. Every line, whether straight or curved, lengthens in the ratio α per degree temperature rise. If you scratch your name on the plate, the line representing your name has the same fractional change in length as any other line. The analogy to a photographic enlargement is shown in Fig. 21-10.

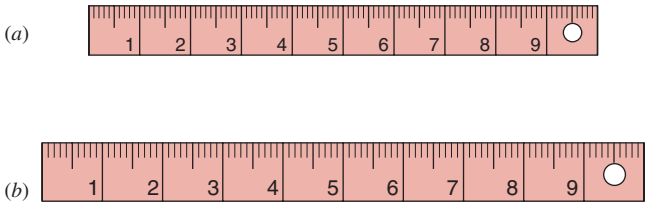

FIGURE 21-10. A steel rule at two different temperatures. The expansion increases in proportion in all dimensions: the scale, the numbers, the hole, and the thickness are all increased by the same factor. (The expansion shown is greatly exaggerated; to obtain such an expansion would require a temperature increase of about 20,000 C°!)

With these ideas in mind, you should be able to show (see Exercises 22 and 23) that to a high degree of accuracy the fractional change in area A per degree temperature change for an isotropic solid is 2α, that is,

$$\Delta A = 2\alpha A\, \Delta T, \tag{21-10}$$

and the fractional change in volume V per degree temperature change for an isotropic solid is 3α, that is,

$$\Delta V = 3\alpha V\, \Delta T. \tag{21-11}$$

Equations 21-8 to 21-11 cannot be applied to the expansion of fluids, because fluids have no definite shape and so the coefficient of linear expansion is not a meaningful quantity for a fluid. Instead, we define the *coefficient of volume expansion* β of a fluid by analogy with Eq. 21-8 or 21-11:

$$\Delta V = \beta V\, \Delta T. \tag{21-12}$$

For liquids, the coefficient of volume expansion is relatively independent of the temperature. Liquids usually expand with increasing temperature (that is, $\beta > 0$). Typical values of β for liquids at room temperature are in the range of 200×10^{-6}/C° to 1000×10^{-6}/C°, more than an order of magnitude larger than the coefficient of volume expansion of most solids (3α from Eq. 21-11). For gases, β is strongly dependent on temperature; in fact, for an ideal gas (discussed in the next section) you can show that $\beta = 1/T$ with T expressed in kelvins (see Exercise 36). For a gas at room temperature and constant pressure, β is about 3300×10^{-6}/C°, as much as an order of magnitude larger than the coefficient of volume expansion for typical liquids.

The most common liquid, water, does not behave like most other liquids. In Fig. 21-11 we show the volume expansion curve for water. Note that above 4°C water expands as the temperature rises, although not linearly. (That is, β is not constant over these large temperature intervals.) As the temperature is lowered from 4°C to 0°C, however, water expands instead of contracting, thus decreasing its density, which is the reason that lakes freeze first at their upper surface. Such an expansion with decreasing temperature is not observed in any other common liquid.

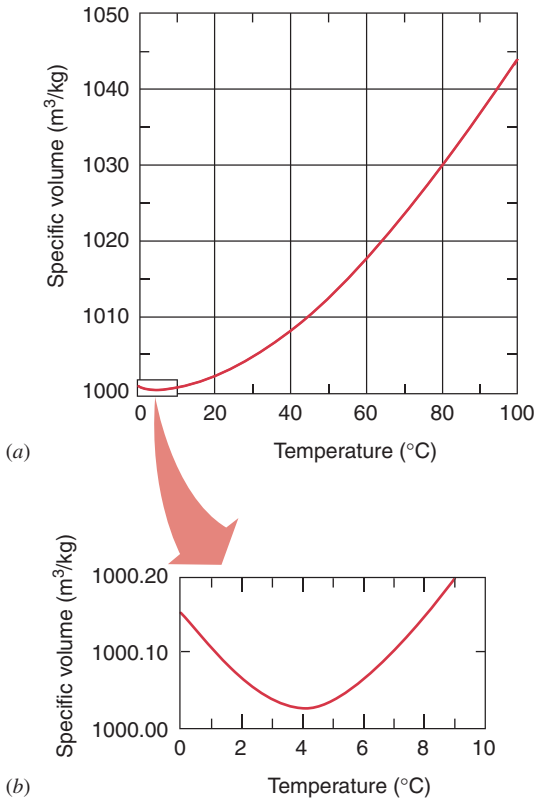

FIGURE 21-11. (*a*) The specific volume (the volume occupied by a particular mass) of water as a function of its temperature. The specific volume is the inverse of the density (the mass per unit volume). (*b*) An enlargement of the region near 4°C, showing a minimum specific volume (or a maximum density).

Microscopic Basis of Thermal Expansion (Optional)

On the microscopic level, thermal expansion of a solid suggests an increase in the average separation between the atoms in the solid. The potential energy curve for two adjacent atoms in a crystalline solid as a function of their internuclear separation is an asymmetric curve like that of Fig. 21-12. As the atoms move close together, their separation decreasing from the equilibrium value r_0, strong repulsive forces come into play, and the potential energy rises steeply ($F = -dU/dr$); as the atoms move farther apart, their separation increasing from the equilibrium value, somewhat weaker attractive forces take over and the potential energy rises more slowly. At a given vibrational energy the separation of the atoms changes periodically from a minimum to a maximum value, the average separation being greater than the equilibrium separation because of the asymmetric nature of the potential energy curve. At still higher vibrational energy the average separation is even greater. The effect is enhanced because, as suggested by Fig. 21-12, the kinetic energy is smaller at larger separations; thus the particles move slower and spend more time at large separations, which then contribute a larger share to the time average.

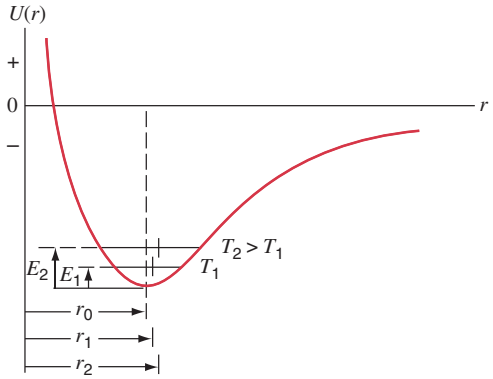

FIGURE 21-12. Potential energy curve for two adjacent atoms in a solid as a function of their internuclear separation distance. The equilibrium separation is r_0. Because the curve is asymmetric, the average separation (r_1, r_2) increases as the temperature (T_1, T_2) and the vibrational energy (E_1, E_2) increase.

Because the vibrational energy increases as the temperature rises, the average separation between atoms increases with temperature, and the entire solid expands.

Note that if the potential energy curve were symmetric about the equilibrium separation, then the average separation would equal the equilibrium separation, no matter how large the amplitude of the vibration. Hence thermal expansion is a direct consequence of the deviation from symmetry of the characteristic potential energy curve of solids.

It should be emphasized that the microscopic models presented here are oversimplifications of a complex phenomenon that can be treated with greater insight using statistical mechanics and quantum theory. ■

21-5 THE IDEAL GAS

Figure 21-5 suggests that real gases such as oxygen, nitrogen, and helium differ from each other as far as the relations among their thermodynamic properties, such as pressure or temperature, are concerned. However, this same figure suggests that, as we examine such real gases at lower and lower densities, their properties seem to converge. That suggests the concept of an *ideal gas*—that is, a gas whose properties represent the limiting behavior of real gases at sufficiently low densities.

The ideal gas is an abstraction, but it is a useful abstraction because (1) real gases—at low enough densities—approximate the behavior of the ideal gas, and (2) the thermodynamic properties of an ideal gas are related to each other in a particularly simple way. Physics is full of useful abstractions and we have met many of them, such as perfectly elastic collisions, massless rods, and unstretchable strings.

Figure 21-13 shows schematically an arrangement with which it is possible to study the properties of real gases and, by extrapolating to sufficiently low densities, to deduce the properties of the ideal gas. An insulated cylinder

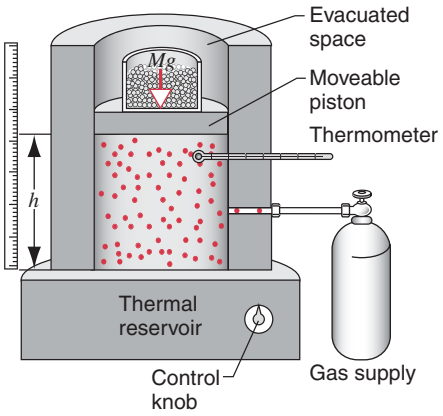

FIGURE 21-13. Gas is confined to a cylinder that is in contact with a thermal reservoir at the (adjustable) temperature T. The piston exerts a total downward force Mg on the gas, which in equilibrium is balanced by the upward force due to the gas pressure. The volume of the gas can be determined from a measurement of the height h of the piston above the bottom of the cylinder, and the temperature of the gas is measured with a suitable thermometer. A gas supply permits additional gas to be added to the cylinder; we assume that a mechanism is also provided for removing gas and for changing the supply to admit different kinds of gas.

that rests on a thermal reservoir (a glorified hot plate) contains a specified quantity of gas, which we can control by adding or removing gas using the gas supply. The temperature of this reservoir—and thus of the gas—can be regulated by turning a control knob. A piston, whose position determines the volume of the gas, can move without friction up and down in the cylinder. Weights, shown here as lead shot, can be added to or removed from the top of the piston, thus determining the pressure exerted by the gas. The variables pressure, volume, temperature, and quantity of gas (number of moles n or number of molecules N) are thus under our control.

From laboratory experiments with real gases, it was found that their pressure p, volume V, and temperature T are related, to a good approximation, by

$$pV = NkT \tag{21-13}$$

Here N is the number of molecules contained in the volume V, and k is a constant called the *Boltzmann constant.* Its measured value is, to three significant figures,

$$k = 1.38 \times 10^{-23}\ \text{J/K}. \tag{21-14}$$

The temperature T in Eq. 21-13 must *always* be expressed in kelvins.

It is often more useful to write Eq. 21-13 in a slightly different form, expressing the quantity of gas not in terms of the number of molecules N but in terms of the number of moles n. (The mole is one of the seven SI base units; see Section 1-5). Either measures the quantity of gas, and they are related by

$$N = nN_A, \tag{21-15}$$

where N_A is the *Avogadro constant*—that is, the number of molecules contained in a mole of any substance. Its value is

$$N_A = 6.02 \times 10^{23}\ \text{molecules/mol}. \tag{21-16}$$

In terms of the number of moles, we can write Eq. 21-13 as

$$pV = nRT, \tag{21-17}$$

where $R = kN_A$ is a new constant, called the *molar gas constant.* Its value is

$$R = 8.31\ \text{J/mol}\cdot\text{K}. \tag{21-18}$$

Equations 21-13 and 21-17 are completely equivalent forms of the *ideal gas law.* This law represents an idealization of the properties of real gases, and it works best as a description of real gases when the pressure and density are low. That is why the lines in Fig. 21-5 representing different gases converged to a single temperature as the pressure (and thus the quantity) of gas was decreased. The ideal gas law also shows why it is critical that the volume of gas in the thermometer of Fig. 21-4 be kept constant, if we want to examine the dependence of pressure on temperature.

In Chapter 22 we explore the ideal gas law by examining the microscopic structure of the gas in terms of the properties of its molecules. It is also possible to "piece together" this law by studying a single relationship between two of the variables in the equation while the others are held constant. Here are three examples of these experiments:

1. The Italian investigator Amadeo Avogadro (1776–1856), for whom the Avogadro constant is named, discovered in 1811 that, under the same conditions of pressure and temperature, equal volumes of different gases contain the same number of molecules ($V \propto N$ for constant p and T). At that time, the very existence of atoms and molecules was much in dispute, and this discovery, known as *Avogadro's law,* was later to provide critical support for the atomic theory.

2. The Anglo-Irish experimenter Robert Boyle (1627–1691) discovered that, if the temperature of a fixed amount of gas is held constant, then the pressure exerted by the gas is inversely proportional to the volume that the gas occupies ($p \propto V^{-1}$ for constant T and N). This observation is known as *Boyle's law.*

3. If the pressure of a fixed quantity of gas is held constant, experiment shows that the volume of the gas is directly proportional to its temperature ($V \propto T$ for constant p and N). These experiments were carried out by the French experimenters Joseph Louis Gay-Lussac (1778–1850) and J.-A.-C. Charles (1746–1823), and this relationship is thus known either as *Gay-Lussac's law* or *Charles' law.*

SAMPLE PROBLEM 21-3. An insulated cylinder fitted with a piston (Fig. 21-13) contains oxygen at a temperature of 20°C and a pressure of 15 atm in a volume of 22 liters. The piston is lowered, decreasing the volume of the gas to 16 liters, and simultaneously the temperature is raised to 25°C. Assuming oxygen to behave like an ideal gas under these conditions, what is the final pressure of the gas?

Solution From Eq. 21-13, since the quantity of gas remains unchanged, we have

$$\frac{p_i V_i}{T_i} = \frac{p_f V_f}{T_f},$$

or

$$p_f = p_i \left(\frac{T_f}{T_i}\right)\left(\frac{V_i}{V_f}\right).$$

Because this is in the form of a ratio, we need not convert p and V into SI units, but *we must express T in absolute (Kelvin) temperature units.* Thus

$$p_f = (15 \text{ atm})\left(\frac{273 + 25 \text{ K}}{273 + 20 \text{ K}}\right)\left(\frac{22 \text{ L}}{16 \text{ L}}\right) = 21 \text{ atm}.$$

MULTIPLE CHOICE

21-1 Temperature and Thermal Equilibrium

1. Consider four objects, *A*, *B*, *C*, and *D*. It is found that *A* and *B* are in thermal equilibrium. It is also found that *C* and *D* are in thermal equilibrium. However, *A* and *C* are *not* in thermal equilibrium. One can conclude that
(A) *B* and *D* are in thermal equilibrium.
(B) *B* and *D* could be in thermal equilibrium, but might not be.
(C) *B* and *D* cannot be in thermal equilibrium.
(D) the zeroth law of thermodynamics does not apply here, because there are more than three objects.

2. Objects *A* and *B* are initially in thermal equilibrium. Objects *A* and *C* are originally not in thermal equilibrium, but the two are placed in thermal contact and quickly reach thermal equilibrium. After doing this
(A) *B* and *C* will also be in thermal equilibrium.
(B) *B* and *C* could be in thermal equilibrium, but might not be.
(C) *B* and *C* cannot be in thermal equilibrium.

21-2 Temperature Scales

3. At what temperature do the Fahrenheit and Celsius scales coincide?
(A) −40°F (B) 0°F (C) 32°F
(D) 40°F (E) 104°F

4. At what temperature do the Fahrenheit and Kelvin scales coincide?
(A) −100°F (B) 273°F (C) 574°F (D) 844°F

21-3 Measuring Temperatures

21-4 Thermal Expansion

5. A large flat slab of metal at temperature T_0 has a hole in it. The metal is warmed to a new temperature $T > T_0$. Upon warming, the area of the hole will
(A) increase. (B) decrease.
(C) remain the same size.
(D) possibly change size, depending on the shape of the hole.

6. Why does a glass sometimes break if you quickly pour boiling water into it?
(A) Hot water expands, pushing the glass out.
(B) The hot water cools when it touches the glass, shrinking and pulling the glass in.
(C) The glass becomes hot and expands, causing the molecules to break.
(D) The inside the glass expands faster than the outside of the glass, causing the glass to break.

7. A mercury-filled glass thermometer is originally at equilibrium in a 20°C water bath. The thermometer is then immersed in a 30°C water bath. The column of mercury in the thermometer will
(A) rise to 30°C and then stop.
(B) first rise above 30°C, then return to 30°C and stop.
(C) first fall below 20°C, then rise to 30°C and stop.
(D) first fall below 20°C, then rise above 30°C, and finally return to 30°C and stop.

8. A strip of copper metal is riveted to a strip of aluminum. The two metals are then heated. What happens?
(A) The strip expands without bending.
(B) The strip expands and bends toward the copper.
(C) The strip expands and bends toward the aluminum.

9. The daily temperature variation of the Golden Gate bridge in San Francisco can be in excess of 20 C°. The bridge is approximately 2 km long and is made of steel (with an asphalt covering on the roadway).
(*a*) What is the approximate change in length of the bridge with this temperature variation?
(A) 4.4 cm (B) 44 cm (C) 4.4 m (D) 44 m
(*b*) If the bridge builders neglected to include expansion joints, then approximately how large of a "bump" would form in the middle of the bridge when it expanded?
(A) 2.1 cm (B) 21 cm (C) 2.1 m (D) 21 m

21-5 The Ideal Gas

10. Which has the higher density (mass per unit volume)—dry air or humid air? Assume that both have the same temperature and pressure.
(A) Dry air (B) Humid air
(C) The densities are the same.

11. Which of the following has the largest particle density (molecules per unit volume)?
(A) 0.8 L of nitrogen gas at 350 K and 100 kPa
(B) 1.0 L of hydrogen gas at 350 K and 150 kPa
(C) 1.5 L of oxygen gas at 300 K and 80 kPa
(D) 2.0 L of helium gas at 300 K and 120 kPa

12. Four different containers each hold 0.5 moles of one of the following gases. Which is at the highest temperature?
(A) 8.0 L of helium gas at 120 kPa
(B) 6.0 L of neon gas at 160 kPa
(C) 4.0 L of argon gas at 250 kPa
(D) 3.0 L of krypton gas at 300 kPa

QUESTIONS

1. Is temperature a microscopic or macroscopic concept?
2. Can we define temperature as a derived quantity, in terms of length, mass, and time? Think of a pendulum, for example.
3. Absolute zero is a minimum temperature. Is there a maximum temperature?
4. Can one object be hotter than another if they are at the same temperature? Explain.
5. Lobster traps are designed so that a lobster can easily get in, but cannot easily get out. Can a diathermic wall be created that allows heat to flow through in one direction only? Explain.
6. Are there physical quantities other than temperature that tend to equalize if two different systems are joined?
7. A piece of ice and a warmer thermometer are suspended in an insulated evacuated enclosure so that they are not in contact. Why does the thermometer reading decrease for a time?
8. What qualities make a particular thermometric property suitable for use in a practical thermometer?
9. What difficulties would arise if you defined temperature in terms of the density of water?
10. Let p_3 be the pressure of the bulb of a constant-volume gas thermometer when the bulb is at the triple-point temperature of 273.16 K and let p be the pressure when the bulb is at room temperature. Given are three constant-volume gas thermometers: for A the gas is oxygen and $p_3 = 20$ cm Hg; for B the gas is also oxygen but $p_3 = 40$ cm Hg; for C the gas is hydrogen and $p_3 = 30$ cm Hg. The measured values of p for the three thermometers are p_A, p_B, and p_C. (*a*) An approximate value of the room temperature T can be obtained with each of the thermometers using

$$T_A = (273.16\text{ K})(p_A/20\text{ cm Hg}),$$

$$T_B = (273.16\text{ K})(p_B/40\text{ cm Hg}),$$

$$T_C = (273.16\text{ K})(p_C/30\text{ cm Hg}).$$

Mark each of the following statements true or false: (1) With the method described, all three thermometers will give the same value of T. (2) The two oxygen thermometers will agree with each other but not with the hydrogen thermometer. (3) Each of the three will give a different value of T. (*b*) In the event that there is a disagreement among the three thermometers, explain how you would change the method of using them to cause all three to give the same value of T.

11. The editor-in-chief of a well-known business magazine, discussing possible warming effects associated with the increasing concentration of carbon dioxide in the Earth's atmosphere (greenhouse effect), wrote: "The polar regions might be three times warmer than now . . ." What do you suppose he meant, and what did he say literally? (See "Warmth and Temperature: A Comedy of Errors," by Albert A. Bartlett, *The Physics Teacher,* November 1984, p. 517.)
12. Although the absolute zero of temperature seems to be experimentally unattainable, temperatures as low as 0.00000002 K have been achieved in the laboratory. Why would physicists strive, as indeed they do, to obtain still lower temperatures? Isn't this low enough for all practical purposes?
13. You put two uncovered pails of water, one containing hot water and one containing cold water, outside in below-freezing weather. The pail with the hot water will usually begin to freeze first. Why? What would happen if you covered the pails?
14. Can a temperature be assigned to a vacuum?
15. Does our "temperature sense" have a built-in sense of direction; that is, does hotter necessarily mean higher temperature, or is this just an arbitrary convention? Celsius, by the way, originally chose the steam point as 0°C and the ice point as 100°C.
16. Many medicine labels inform the user to store below 86°F. Why 86? (Hint: Change to Celsius.) (See *The Science Almanac,* 1985–1986, p. 430.)
17. How would you suggest measuring the temperature of (*a*) the Sun, (*b*) the Earth's upper atmosphere, (*c*) an insect, (*d*) the Moon, (*e*) the ocean floor, and (*f*) liquid helium?
18. Considering the Celsius, Fahrenheit, and Kelvin scales, does any one stand out as "nature's scale"? Discuss.
19. Is one gas any better than another for purposes of a standard constant-volume gas thermometer? What properties are desirable in a gas for such purposes?
20. State some objections to using water-in-glass as a thermometer. Is mercury-in-glass an improvement? If so, explain why.
21. What are the dimensions of α, the coefficient of linear expansion? Does the value of α depend on the unit of length used? When Fahrenheit degrees are used instead of Celsius degrees as the unit of temperature change, does the numerical value of α change? If so, how? If not, prove it.
22. A metal ball can pass through a metal ring. When the ball is heated, however, it gets stuck in the ring. What would happen if the ring, rather than the ball, were heated?
23. A bimetallic strip, consisting of two different metal strips riveted together, is used as a control element in the common thermostat. Explain how it works.
24. Two strips, one of iron and one of zinc, are riveted together side by side to form a straight bar that curves when heated. Why is the iron on the inside of the curve?
25. Explain how the period of a pendulum clock can be kept constant with temperature by attaching vertical tubes of mercury to the bottom of the pendulum.
26. Why should a chimney be freestanding—that is, not part of the structural support of the house?
27. Water expands when it freezes. Can we define a coefficient of volume expansion for the freezing process?
28. Explain why the apparent expansion of a liquid in a glass bulb does not give the true expansion of the liquid.
29. Does the change in volume of an object when its temperature is raised depend on whether the object has cavities inside, other things being equal?
30. Why is it much more difficult to make a precise determination of the coefficient of expansion of a liquid than of a solid?
31. A common model of a solid assumes the atoms to be points executing simple harmonic motion about mean lattice positions. What would be the coefficient of linear expansion of such a lattice?
32. Explain the fact that the temperature of the ocean at great depths is very constant the year round, at a temperature of about 4°C.
33. Explain why lakes freeze first at the surface.

34. What causes water pipes to burst in the winter?

35. What can you conclude about how the melting point of ice depends on pressure from the fact that ice floats on water?

36. Two equal-size rooms communicate through an open doorway. However, the average temperatures in the two rooms are maintained at different values. In which room is there more air?

37. It is found that the weight of an empty, flat, thin plastic bag is not changed when the bag is filled with air. Why not?

38. Why does smoke rise, rather than fall, from a lighted candle?

39. Do the pressure and volume of air in a house change when the furnace raises the temperature significantly? If not, is the ideal gas law violated?

EXERCISES

21-1 Temperature and Thermal Equilibrium

21-2 Temperature Scales

1. The boiling point and the melting point for water on the Fahrenheit scale were chosen so the difference between the two temperatures would be 180 F°, a number that is evenly divisible by 2, 3, 4, 5, 6, and 9. Devise a new temperature scale S so that absolute zero is 0°S and $T_{\text{bp, water}} - T_{\text{mp, water}} = 180$ S°. (*a*) What is the conversion formula from Celsius to S? (*b*) What are $T_{\text{bp, water}}$ and $T_{\text{mp, water}}$ in S?

2. Absolute zero is -273.15°C. Find absolute zero on the Fahrenheit scale.

3. Repeat Exercise 1, except choose the new temperature scale Q so that absolute zero is 0°Q and $T_{\text{bp, water}} - T_{\text{mp, water}} = 100$ Q°. (*a*) What is the conversion formula from Celsius to Q? (*b*) What is $T_{\text{bp, water}}$ and $T_{\text{mp, water}}$ in Q? (*c*) This scale actually exists. What is the official name?

4. (*a*) The temperature of the surface of the Sun is about 6000 K. Express this on the Fahrenheit scale. (*b*) Express normal human body temperature, 98.6°F on the Celsius scale. (*c*) In the continental United States, the lowest officially recorded temperature is -70°F at Rogers Pass, Montana. Express this on the Celsius scale. (*d*) Express the normal boiling point of oxygen, -183°C, on the Fahrenheit scale. (*e*) At what Celsius temperature would you find a room to be uncomfortably warm?

5. If your doctor tells you that your temperature is 310 K, should you worry? Explain your answer.

6. At what temperature is the Fahrenheit scale reading equal to (*a*) twice that of the Celsius and (*b*) half that of the Celsius?

21-3 Measuring Temperatures

7. A *resistance thermometer* is a thermometer in which the electrical resistance changes with temperature. We are free to define temperatures measured by such a thermometer in kelvins (K) to be directly proportional to the resistance R, measured in ohms (Ω). A certain resistance thermometer is found to have a resistance R of 90.35 Ω when its bulb is placed in water at the triple-point temperature (273.16 K). What temperature is indicated by the thermometer if the bulb is placed in an environment such that its resistance is 96.28 Ω?

8. A thermocouple is formed from two different metals, joined at two points in such a way that a small voltage is produced when the two junctions are at different temperatures. In a particular iron–constantan thermocouple, with one junction held at 0°C, the output voltage varies linearly from 0 to 28.0 mV as the temperature of the other junction is raised from 0 to 510°C. Find the temperature of the variable junction when the thermocouple output is 10.2 mV.

9. The amplification or *gain* of a transistor amplifier may depend on the temperature. The gain for a certain amplifier at room temperature (20.0°C) is 30.0, whereas at 55.0°C it is 35.2. What would the gain be at 28.0°C if the gain depends linearly on temperature over this limited range?

10. If the gas temperature at the steam point is 373.15 K, what is the limiting value of the ratio of the pressures of a gas at the steam point and at the triple point of water when the gas is kept at constant volume?

11. Two constant-volume gas thermometers are assembled, one using nitrogen as the working gas and the other using helium. Both contain enough gas so that $p_{\text{tr}} = 100$ cm Hg. What is the difference between the pressures in the two thermometers if both are inserted into a water bath at the boiling point? Which pressure is the higher of the two? See Fig. 21-5.

21-4 Thermal Expansion

12. An aluminum flagpole is 33 m high. By how much does its length increase as the temperature increases by 15 C°?

13. The Pyrex glass mirror in the telescope at the Mount Palomar Observatory (the Hale telescope) has a diameter of 200 in. The most extreme temperatures ever recorded on Palomar Mountain are -10°C and 50°C. Determine the maximum change in the diameter of the mirror.

14. A circular hole in an aluminum plate is 2.725 cm in diameter at 12°C. What is its diameter when the temperature of the plate is raised to 140°C?

15. Steel railroad tracks are laid when the temperature is -5.0°C. A standard section of rail is then 12.0 m long. What gap should be left between rail sections so that there is no compression when the temperature gets as high as 42°C?

16. A glass window is 200 cm by 300 cm at 10°C. By how much has its area increased when its temperature is 40°C? Assume that the glass is free to expand.

17. A brass cube has an edge length of 33.2 cm at 20.0°C. Find (*a*) the increase in surface area and (*b*) the increase in volume when it is heated to 75.0°C.

18. What is the volume of a lead ball at -12°C if its volume at 160°C is 530 cm^3?

19. (*a*) From the graph of Fig. 21-11, estimate the coefficient of volume expansion for water at room temperature (20°C). (*b*) What is the coefficient of volume expansion near 4°C?

20. Soon after the Earth formed, heat released by the decay of radioactive elements raised the average internal temperature from 300 to 3000 K, at about which value it remains today. Assuming an average coefficient of volume expansion of $3.2 \times 10^{-5}\ \text{K}^{-1}$, by how much has the radius of the Earth increased since its formation?

21. A rod is measured to be 20.05 cm long using a steel ruler at a room temperature of 20°C. Both the rod and the ruler are placed in an oven at 270°C, where the rod now measures 20.11 cm using the same rule. Calculate the coefficient of thermal expansion for the material of which the rod is made.

22. The area A of a rectangular plate is ab. Its coefficient of linear expansion is α. After a temperature rise ΔT, side a is longer by Δa and side b is longer by Δb. Show that if we neglect the small quantity $\Delta a\,\Delta b/ab$ (see Fig. 21-14), then $\Delta A = 2\alpha A\,\Delta T$, verifying Eq. 21-10.

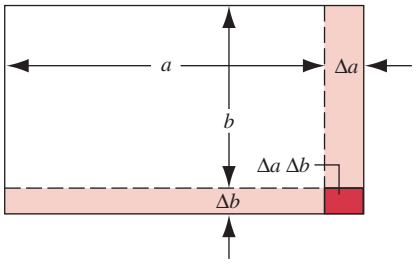

FIGURE 21-14. Exercise 22.

23. Prove that, if we neglect extremely small quantities, the change in volume of a solid upon expansion through a temperature rise ΔT is given by $\Delta V = 3\alpha V\Delta T$, where α is the coefficient of linear expansion. See Eq. 21-11.

24. When the temperature of a copper penny (which is not pure copper) is raised by 100 C°, its diameter increases by 0.18%. Find the percent increase in (*a*) the area of a face, (*b*) the thickness, (*c*) the volume, and (*d*) the mass of the penny. (*e*) Calculate its coefficient of linear expansion.

25. Density is mass divided by volume. If the volume V is temperature dependent, so is the density ρ. Show that the change in density $\Delta\rho$ with change in temperature ΔT is given by

$$\Delta\rho = -\beta\rho\,\Delta T,$$

where β is the coefficient of volume expansion. Explain the minus sign.

26. When the temperature of a metal cylinder is raised from 60 to 100°C, its length increases by 0.092%. (*a*) Find the percent change in density. (*b*) Identify the metal.

27. A steel rod is 3.000 cm in diameter at 25°C. A brass ring has an interior diameter of 2.992 cm at 25°C. At what common temperature will the ring just slide onto the rod?

28. A composite bar of length $L = L_1 + L_2$ is made from a bar of material 1 and length L_1 attached to a bar of material 2 and length L_2 as shown in Fig. 21-15. (*a*) Show that the effective coefficient of linear expansion α for this bar is given by $\alpha = (\alpha_1 L_1 + \alpha_2 L_2)/L$. (*b*) Using steel and brass, design such a composite bar whose length is 52.4 cm and whose effective coefficient of linear expansion is 13×10^{-6}/C°.

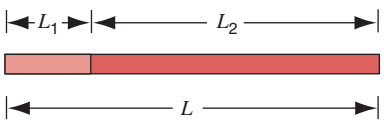

FIGURE 21-15. Exercise 28.

29. At 100°C a glass flask is completely filled by 891 g of mercury. What mass of mercury is needed to fill the flask at −35°C? (The coefficient of linear expansion of glass is 9.0×10^{-6}/C°; the coefficient of volume expansion of mercury is 1.8×10^{-4}/C°.)

30. (*a*) Prove that the change in rotational inertia I with temperature of a solid object is given by $\Delta I = 2\alpha I\,\Delta T$. (*b*) A thin uniform brass rod, spinning freely at 230 rev/s about an axis perpendicular to it at its center, is heated without mechanical contact until its temperature increases by 170 C°. Calculate the change in angular velocity.

31. A cylinder placed in frictionless bearings is set rotating about its axis. The cylinder is then heated, without mechanical contact, until its radius is increased by 0.18%. What is the percent change in the cylinder's (*a*) angular momentum, (*b*) angular velocity, and (*c*) rotational energy?

32. (*a*) Prove that the change in period P of a physical pendulum with temperature is given by $\Delta P = \frac{1}{2}\alpha P\Delta T$. (*b*) A clock pendulum made of invar has a period of 0.500 s and is accurate at 20°C. If the clock is used in a climate where the temperature averages 30°C, what approximate correction to the time given by the clock is necessary at the end of 30 days?

33. A pendulum clock with a pendulum made of brass is designed to keep accurate time at 20°C. How much will the error be, in seconds per hour, if the clock operates at 0°C?

34. An aluminum cup of 110 cm^3 capacity is filled with glycerin at 22°C. How much glycerin, if any, will spill out of the cup if the temperature of the cup and glycerin is raised to 28°C? (The coefficient of volume expansion of glycerin is 5.1×10^{-4}/C°.)

35. A 1.28-m-long vertical glass tube is half-filled with a liquid at 20.0°C. How much will the height of the liquid column change when the tube is heated to 33.0°C? Assume that $\alpha_{glass} = 1.1 \times 10^{-5}$/C° and $\beta_{liquid} = 4.2 \times 10^{-5}$/C°.

21-5 The Ideal Gas

36. (*a*) Using the ideal gas law and the definition of the coefficient of volume expansion (Eq. 21-12), show that $\beta = 1/T$ for an ideal gas at constant pressure. (*b*) In what units must T be expressed? If T is expressed in those units, can you express β in units of $(C°)^{-1}$? (*c*) Estimate the value of β for an ideal gas at room temperature.

37. (*a*) Calculate the volume occupied by 1.00 mol of an ideal gas at standard conditions—that is, pressure of 1.00 atm (= 1.01×10^5 Pa) and temperature of 0°C (= 273 K). (*b*) Show that the number of molecules per cubic centimeter (the *Loschmidt number*) at standard conditions is 2.68×10^{19}.

38. The best vacuum that can be attained in the laboratory corresponds to a pressure of about 10^{-18} atm, or 1.01×10^{-13} Pa. How many molecules are there per cubic centimeter in such a vacuum at 22°C?

39. A quantity of ideal gas at 12.0°C and a pressure of 108 kPa occupies a volume of 2.47 m^3. (*a*) How many moles of the gas are present? (*b*) If the pressure is now raised to 316 kPa and the temperature is raised to 31.0°C, how much volume will the gas now occupy? Assume there are no leaks.

40. Oxygen gas having a volume of 1130 cm^3 at 42.0°C and a pressure of 101 kPa expands until its volume is 1530 cm^3 and its pressure is 106 kPa. Find (*a*) the number of moles of oxygen in the system and (*b*) its final temperature.

41. An automobile tire has a volume of 988 in.^{3} and contains air at a gauge pressure of 24.2 lb/in.^{2} where the temperature is -2.60°C. Find the gauge pressure of the air in the tire when its temperature rises to 25.6°C and its volume increases to 1020 in.^{3}. (Hint: It is not necessary to convert from British to SI units. Why? Use $p_{atm} = 14.7$ lb/in.^{2}.)

42. Estimate the mass of the Earth's atmosphere. Express your estimate as a fraction of the mass of the Earth. Recall that atmospheric pressure equals 101 kPa.

43. An air bubble of 19.4 cm^3 volume is at the bottom of a lake 41.5 m deep where the temperature is 3.80°C. The bubble rises to the surface, which is at a temperature of 22.6°C. Take the temperature of the bubble to be the same as that of the surrounding water and find its volume just before it reaches the surface.

44. An open–closed pipe of length $L = 25.0$ m contains air at atmospheric pressure. It is thrust vertically into a freshwater lake until the water rises halfway up in the pipe, as shown in Fig. 21-16. What is the depth h of the lower edge of the pipe? Assume that the temperature is the same everywhere and does not change.

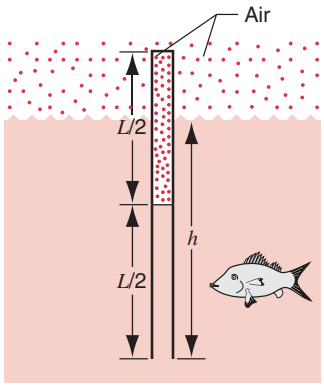

FIGURE 21-16. Exercise 44.

PROBLEMS

1. It is an everyday observation that hot and cold objects cool down or warm up to the temperature of their surroundings. If the temperature difference ΔT between an object and its surroundings ($\Delta T = T_{obj} - T_{sur}$) is not too great, the rate of cooling or warming of the object is proportional, approximately, to this temperature difference; that is,

$$\frac{d\Delta T}{dt} = -A(\Delta T),$$

where A is a constant. The minus sign appears because ΔT decreases with time if ΔT is positive and increases if ΔT is negative. This is known as *Newton's law of cooling*. (*a*) On what factors does A depend? What are its dimensions? (*b*) If at some instant $t = 0$ the temperature difference is ΔT_0, show that it is

$$\Delta T = \Delta T_0 e^{-At}$$

at a time t later.

2. Early in the morning the heater of a house breaks down. The outside temperature is -7.0°C. As a result, the inside temperature drops from 22 to 18°C in 45 min. How much longer will it take for the inside temperature to fall by another 4.0 C°? Assume that the outside temperature does not change and that Newton's law of cooling applies; see Problem 1.

3. Show that when the temperature of a liquid in a barometer changes by ΔT, and the pressure is constant, the height h changes by $\Delta h = \beta h \Delta T$, where β is the coefficient of volume expansion of the liquid. Neglect the expansion of the glass tube.

4. A particular gas thermometer is constructed of two gas-containing bulbs, each of which is put into a water bath, as shown in Fig. 21-17. The pressure difference between the two bulbs is measured by a mercury manometer as shown in the figure. Appropriate reservoirs, not shown in the diagram, maintain constant gas volume in the two bulbs. There is no difference in pressure when both baths are at the triple point of water. The pressure difference is 120 mm Hg when one bath is at the triple point and the other is at the boiling point of water. Finally, the pressure difference is 90.0 mm Hg when one bath is at the triple point and the other is at an unknown temperature to be measured. Find the unknown temperature.

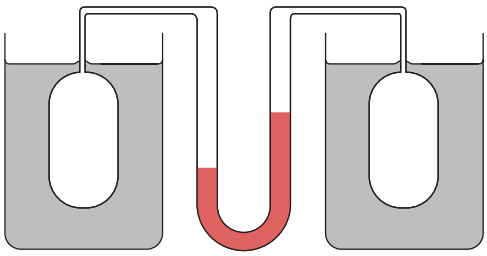

FIGURE 21-17. Problem 4.

5. Show that if α is dependent on the temperature T, then

$$L \approx L_0 \left[1 + \int_{T_0}^{T} \alpha(T)\, dT\right],$$

where L_0 is the length at the reference temperature T_0.

6. In a certain experiment, it was necessary to be able to move a small radioactive source at selected, extremely slow speeds. This was accomplished by fastening the source to one end of an aluminum rod and heating the central section of the rod in a controlled way. If the effective heated section of the rod in Fig. 21-18 is 1.8 cm, at what constant rate must the temperature of the rod be made to change if the source is to move at a constant speed of 96 nm/s?

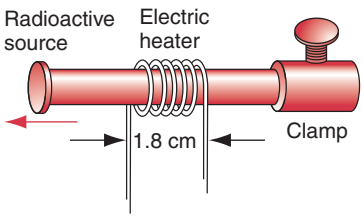

FIGURE 21-18. Problem 6.

7. (*a*) Show that if the lengths of two rods of different solids are inversely proportional to their respective coefficients of linear expansion at the same initial temperature, the difference in length between them will be constant at all temperatures. (*b*) What should be the lengths of a steel and a brass rod at 0°C so that at all temperatures their difference in length is 0.30 m?

8. As a result of a temperature rise of 32 C°, a bar with a crack at its center buckles upward, as shown in Fig. 21-19. If the fixed distance $L_0 = 3.77$ m and the coefficient linear expansion is 25×10^{-6}/C°, find x, the distance to which the center rises.

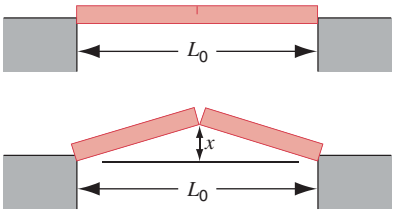

FIGURE 21-19. Problem 8.

9. Figure 21-20 shows the variation of the coefficient of volume expansion of water between 4°C and 20°C. The density of water at 4°C is 1000 kg/m^3. Calculate the density of water at 20°C.

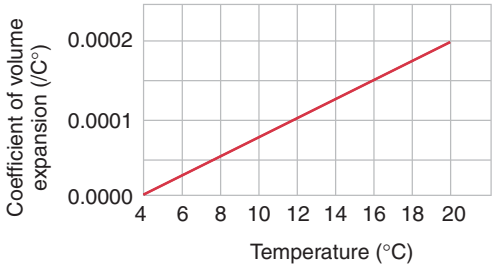

FIGURE 21-20. Problem 9.

10. Consider a mercury-in-glass thermometer. Assume that the cross section of the capillary is constant at A and that V is the volume of the bulb of mercury at 0.00°C. Suppose that the mercury just fills the bulb at 0.00°C. Show that the length L of the mercury column in the capillary at a temperature T, in °C, is

$$L = \frac{V}{A}(\beta - 3\alpha)T,$$

that is, proportional to the temperature, where β is the coefficient of volume expansion of mercury and α is the coefficient of linear expansion of glass.

11. Three equal-length straight rods, of aluminum, invar, and steel, all at 20°C, for an equilateral triangle with hinge pins at the vertices. At what temperature will the angle opposite the invar rod be 59.95°? See Appendix I for needed trigonometric formulas.

12. A glass tube nearly filled with mercury is attached in tandem to the bottom of an iron pendulum rod 100 cm long. How high must the mercury be in the glass tube so that the center of mass of this pendulum will not rise or fall with changes in temperature? (The cross-sectional area of the tube is equal to that of the iron rod. Neglect the mass of the glass. Iron has a density of 7.87×10^3 kg/m^3 and a coefficient of linear expansion equal to 12×10^{-6}/C°. The coefficient of volume expansion of mercury is 18×10^{-5}/C°.)

13. An aluminum cube 20 cm on an edge floats on mercury. How much farther will the block sink when the temperature rises from 270 to 320 K? (The coefficient of volume expansion of mercury is 1.8×10^{-4}/C°.)

14. Dumet wire was developed to allow for the expansion of glass in lightbulbs. The wire consists of a core of nickel–steel (invar) surrounded by a sheath of copper. The diameters of the core and of the sheath are chosen so that the wire duplicates the expansion characteristics of glass. (*a*) Show that the ratio of the nickel–steel radius to that of the copper sheath should be

$$\frac{r_{\text{nickel-steel}}}{r_{\text{copper}}} = \sqrt{\frac{\alpha_{\text{copper}} - \alpha_{\text{glass}}}{\alpha_{\text{copper}} - \alpha_{\text{nickel-steel}}}}$$

(*b*) What is a typical value for this ratio?

15. The distance between the towers of the main span of the Golden Gate Bridge near San Francisco is 4200 ft (Fig. 21-21). The sag of the cable halfway between the towers at 50°F is 470 ft. Take $\alpha = 6.5 \times 10^{-6}$/F° for the cable and compute (*a*) the change in length of the cable and (*b*) the change in sag for a temperature change from 10 to 90°F. Assume no bending or separation of the towers and a parabolic shape for the cable.

FIGURE 21-21. Problem 15.

16. A weather balloon is loosely inflated with helium at a pressure of 1.00 atm (= 76.0 cm Hg) and a temperature of 22.0°C. The gas volume is 3.47 m^3. At an elevation of 6.50 km, the atmospheric pressure is down to 36.0 cm Hg, and the helium has expanded, being under no restraint from the confining bag. At this elevation the gas temperature is −48.0°C. What is the gas volume now?

17. Two vessels of volumes 1.22 L and 3.18 L contain krypton gas and are connected by a thin tube. Initially, the vessels are at the same temperature, 16.0°C, and the same pressure, 1.44 atm. The larger vessel is then heated to 108°C while the smaller one remains at 16.0°C. Calculate the final pressure. (Hint: There are no leaks.)

18. Container A contains an ideal gas at a pressure of 5.0×10^5 Pa and at a temperature of 300 K. It is connected by a thin tube to container B with four times the volume of A; see Fig. 21-22. B contains the same ideal gas at a pressure of 1.0×10^5 Pa and at a temperature of 400 K. The connecting valve is opened, and equilibrium is achieved at a common pressure while the temperature of each container is kept constant at its initial value. What is the final pressure in the system?

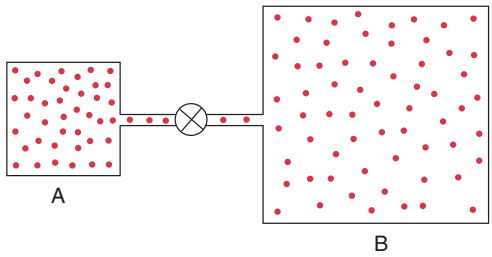

FIGURE 21-22. Problem 18.

19. The variation in pressure in the Earth's atmosphere, assumed to be at a uniform temperature, is given by $p = p_0 e^{-Mgy/RT}$, where M is the molar mass of the air. (See Section 15-3.) Show that $n_V = n_{V,0} e^{-Mgy/RT}$, where n_V is the number of molecules per unit volume.

20. A soap bubble of radius $r_0 = 2.0$ mm floats freely inside a vacuum bell jar. The pressure inside the bell jar is originally $p = 1$ atm. The vacuum pump is turned on and the pressure in the bell jar is slowly decreased to zero while the temperature of the gas inside the bubble remains constant. What is the radius of the soap bubble when the outside pressure drops to zero? The surface tension for a soap bubble is $\gamma = 2.50 \times 10^{-2}$ N/m. (See Computer Problem 1.)

21. A mercury-filled manometer with two unequal-length arms of the same cross-sectional area is sealed off with the same pressure p in the two arms, as in Fig. 21-23. With the temperature constant, an additional 10.0 cm^3 of mercury is admitted through the stopcock at the bottom. The level on the left increases 6.00 cm and that on the right increases 4.00 cm. Find the pressure p.

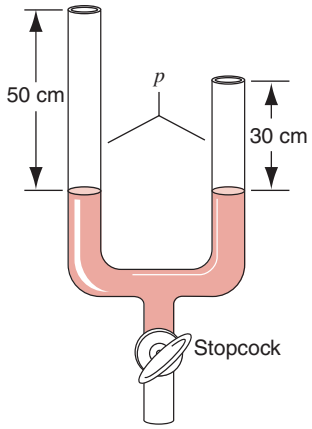

FIGURE 21-23. Problem 21.

22. The "surface tension" of a certain spherical balloon is proportional to the radius of the balloon. Originally the balloon is filled with 10.0 L of an ideal gas at 80°C and 103 kPa. The gas cools to 20°C; find the new volume of the balloon. Assume that the pressure outside the balloon remains at 101 kPa.

COMPUTER PROBLEMS

1. A soap bubble with surface tension $\gamma = 2.50 \times 10^{-2}$ N/m has a radius $r_0 = 2.0$ mm when the pressure outside the bubble is 1.0 atmosphere. (*a*) Numerically calculate the radius of the soap bubble when the pressure outside the bubble drops to 0.5 atm. (*b*) Numerically calculate the radius of the soap bubble if the pressure outside the bubble is raised to 2.0 atm.

2. A small balloon is filled with nitrogen gas (assumed ideal) at the bottom of the Marianas Trench, 35,000 ft beneath the surface of the ocean. The balloon originally has a radius of 1.0 mm, is massless, and is infinitely expandable without any surface tension, but always keeps a spherical shape. Assume the ideal gas inside the balloon is at 4°C throughout this problem. The balloon begins to rise to the surface, as the balloon rises it expands, and as it moves there is a retarding force f proportional to speed v and balloon radius r given by

$$f = 6\pi\eta r v,$$

where $\eta = 1.7 \times 10^{-3}$ N · s/m is the viscosity of water. (*a*) Calculate the initial buoyant force on the balloon. (*b*) What will be the size of the balloon on the surface? (*c*) Numerically solve this problem to find out how long it takes for the balloon to rise to the surface.